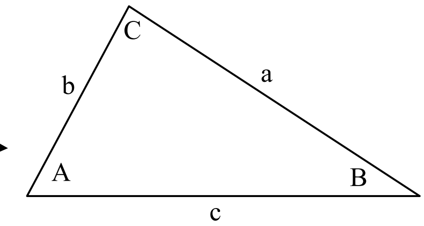
Trigonometry - Activity 21

General Triangle Solution: Given three sides.



When the three side lengths 'a', 'b' and 'c' of a triangle are known, then the three internal angles 'A', 'B' and 'C' can be found. (See diagram at right.)

These are the formulas used to solve this type of problem:

The **sum of the internal angles** equals 180º ...

$$A + B + C = 180^\circ$$

The '**sine rule**' ...

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The '**cosine rule**' ...

$$a^2 = b^2 + c^2 - 2bc\cos A$$

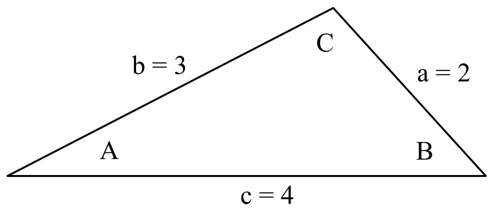
or

$$b^2 = a^2 + c^2 - 2ac\cos B$$

or

$$c^2 = b^2 + a^2 - 2ba\cos C$$

We will demonstrate the procedure for solving a triangle given three sides, using this triangle as an example:



1. Use the cosine rule to find the **largest** angle. (Which is opposite the largest side.)

$$c^2 = b^2 + a^2 - 2ba\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$= \frac{2^2 + 3^2 - 4^2}{2 \times 2 \times 3}$$

$$= -0.25$$

Find the inverse cos of -0.25 using a scientific calculator...

$$C = \cos^{-1}(-0.25)$$
$$= 104.478^\circ$$

2. Use the sine rule to find one of the remaining angles.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\sin A = \frac{a \sin C}{c}$$

$$= \frac{2\sin 104.478^\circ}{4}$$

$$= 0.484123$$

Find the inverse sin of 0.484123 using a scientific calculator...

$$A = \sin^{-1}(0.484123)$$
$$= 28.955^\circ$$

3. Use the 'sum of internal angles' rule to find the remaining angle.

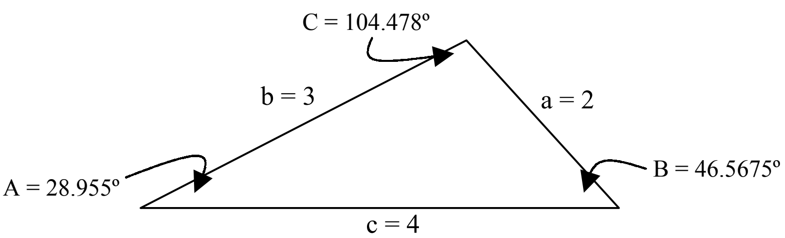
The sum of the internal angles equals 180º ...

$A + B + C = 180^\circ$

so

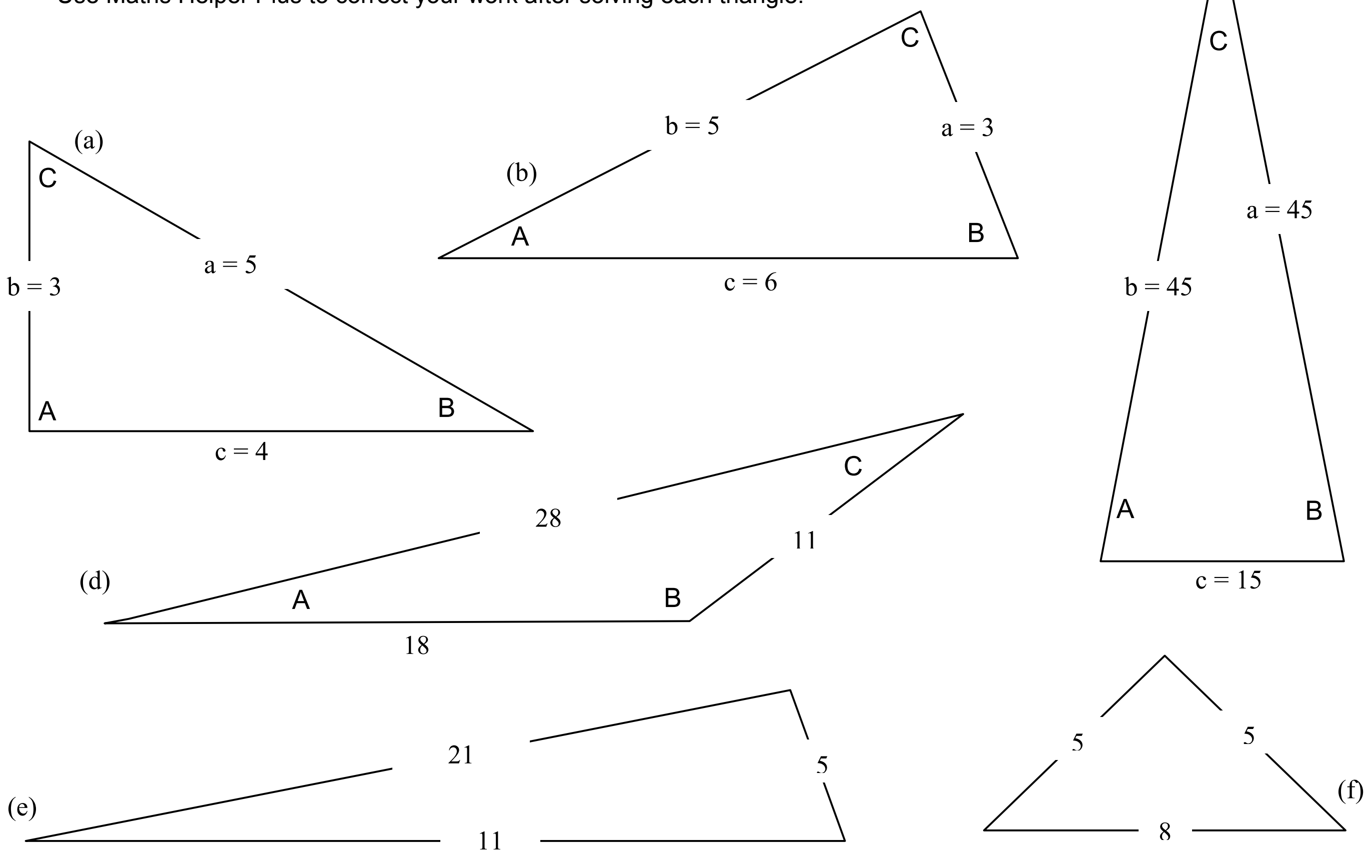
B = 180º – (A+C)
= 180º – (28.955º + 104.478º)
= 180º – 133.433º
= 46.5675º

The triangle is now solved. This diagram shows all of the sides and angles:



NOTE: There can only be one angle in a triangle that is obtuse (greater than 90°). If a triangle has an obtuse angle, then it will be opposite the largest side. The reason for finding it first is that in the next step we will use the sine rule to find the second angle. The inverse sin operation that we will use can only give us acute angles (less than 90°), so we avoid a possible wrong answer by first eliminating the only possibility of an obtuse angle.

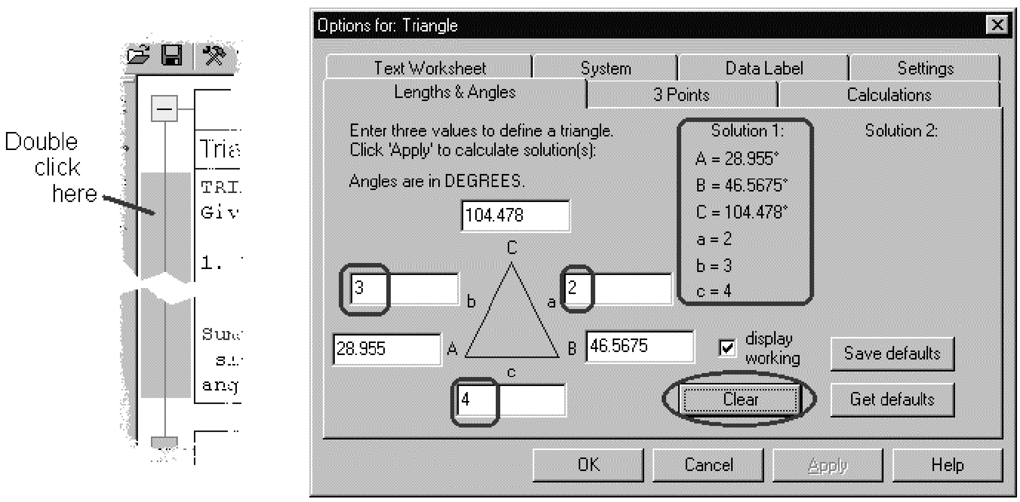
1) Six triangles are shown below. For each triangle, find the three interior angles. Show all working steps.
Use Maths Helper Plus to correct your work after solving each triangle.



To correct your work...

2) Start Maths Helper Plus and load the 'R2 - Triangle_sover_SSS.mhp' document.
This document shows the working steps and a diagram for solving triangles when given three side lengths.

3) Display the triangle solver options box
Double click the mouse in the border to the left of the calculations. (This area is shaded in the diagram below.)
The triangle solver options box will display its 'Lengths & Angles' tab...



Click the 'Clear' button to remove the previous triangle, then click on the 'a' edit box. Now type the length for side 'a' of your triangle. Repeat for 'b' and 'c'.

Click the 'Apply' button at the bottom of the edit box. The calculated values will display on the options box.

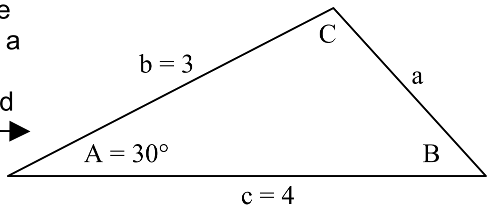
Click the 'OK' button to close the options box. The calculations and triangle diagram will be displayed on your screen.

NOTE: **If the diagram becomes too big** for your computer screen, press the 'F10' key to make it smaller.
To make the diagram bigger, hold down 'Shift' while you press 'F10'.

Trigonometry - Activity 22

General Triangle Solution: Given two sides and the included angle.

The angle between two sides of a triangle is called the included angle of those sides. If any two sides and the included angle are known for a triangle, then the remaining side length and the two unknown angles can be found. In the diagram (right), the two given sides are b = 3 and c = 4. The included angle between these sides is A = 30°.



These are the formulas used to solve this type of problem:

The **sum of the internal angles** equals 180º ...

$$A + B + C = 180^\circ$$

The '**sine rule**' ...

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The '**cosine rule**' ...

$$a^2 = b^2 + c^2 - 2bc \cos A$$

or

$$b^2 = a^2 + c^2 - 2ac \cos B$$

or

$$c^2 = b^2 + a^2 - 2ba \cos C$$

We will demonstrate the procedure for solving a triangle given two sides and the included angle, using the triangle given above. These are the steps:

1. Use the cosine rule to find the unknown side.

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$= 3^2 + 4^2 2 \times 3 \times 4 \cos 30°$$
$$= 4.215390309$$

Taking the positive square root...

$a = 2.05314$

2. Use the sine rule to find the smaller of the two unknown angles. (A smaller angle will be opposite a smaller side.)

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\sin B = \frac{b \sin A}{a}$$
$$= \frac{3 \sin 30°}{2.05314}$$
$$= 0.730588$$

Find the inverse sin of 0.730588 using a scientific calculator...

$A = \sin^{-1}(0.730588)$
$= 46.9357°$

3. Use the 'sum of internal angles' rule to find the remaining angle.

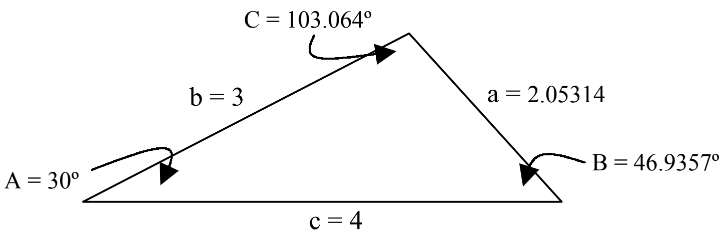
The sum of the internal angles equals 180º ...

$A + B + C = 180°$

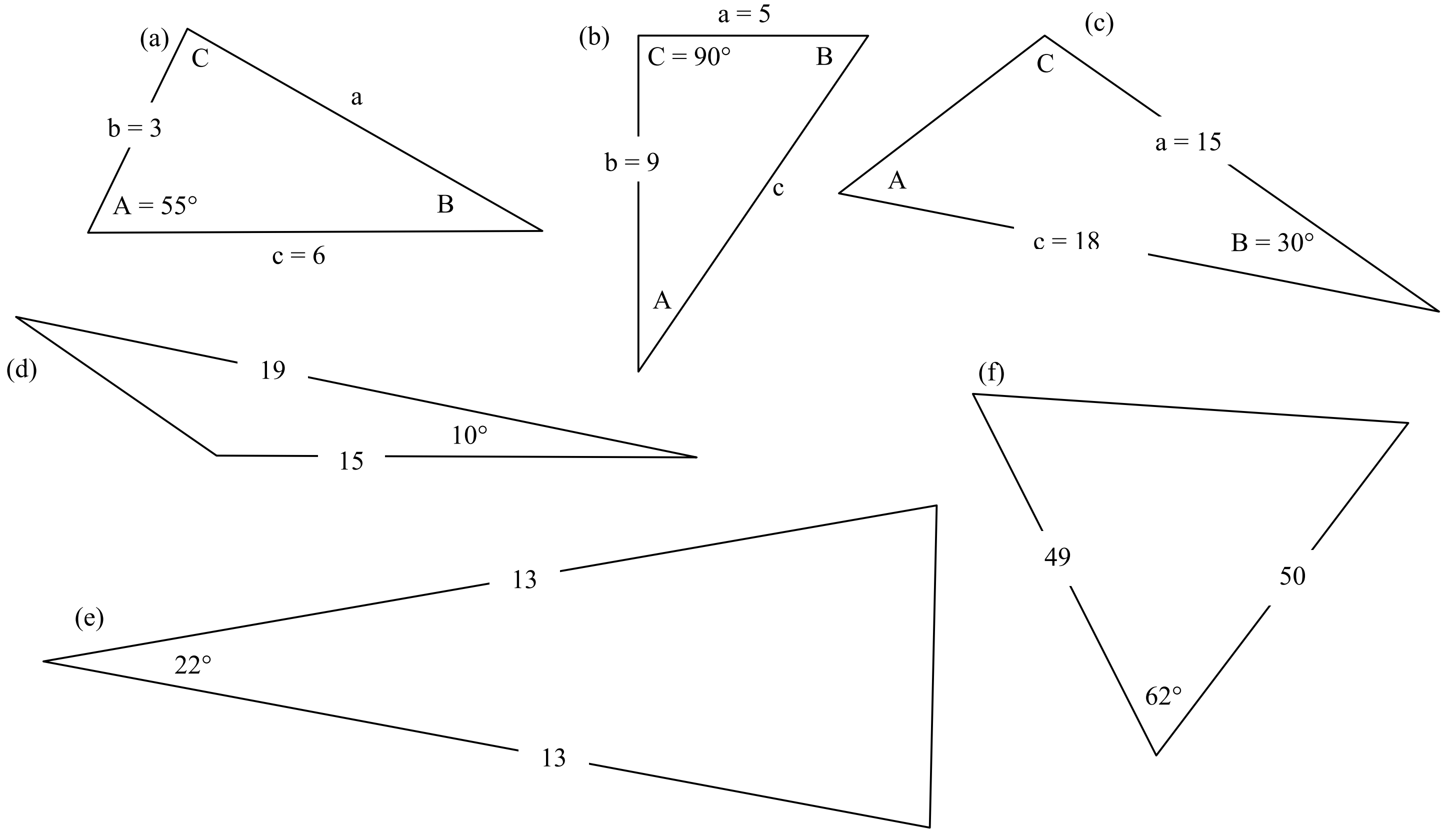
so

$C = 180° - (A+B)$
$= 180° - (30° + 46.9357°)$
$= 180° - 76.9357°$
$= 103.064°$

The triangle is now solved. This diagram shows all of the sides and angles:



1) Six triangles are shown below. For each triangle, find the three interior angles using the method described on the front of this sheet. Show all working steps. Use Maths Helper Plus to correct your work after solving each triangle.



To correct your work...

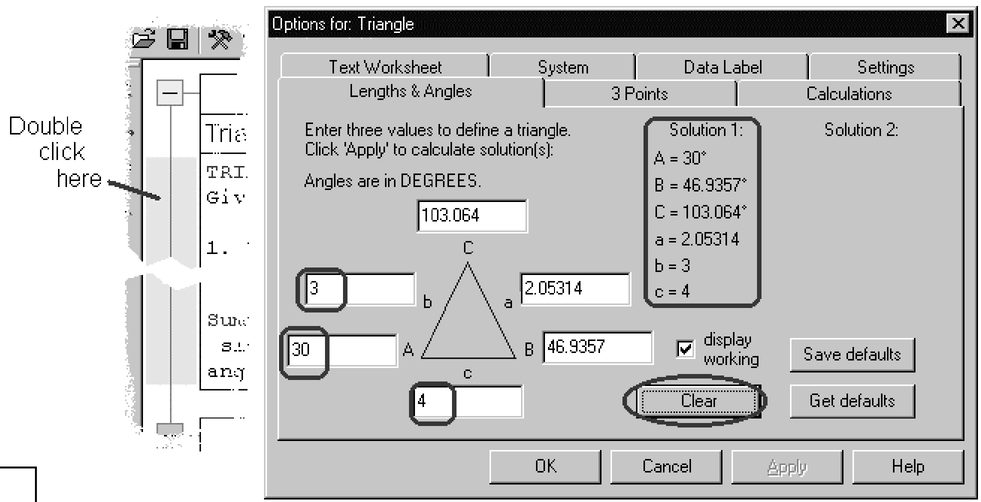
2) Start Maths Helper Plus and load the 'R2 - Triangle_sover_SAS.mhp' document.
This document shows the working steps and a diagram for solving triangles given two sides and the included angle.

3) Display the triangle solver options box
Double click the mouse in the border to the left of the calculations. (This area is shaded in the diagram below.)
The triangle solver options box will display its 'Lengths & Angles' tab (See below) ...

Click the 'Clear' button to remove the previous triangle, then enter your given angle and two sides. You can enter these in three ways, but the answer will be correct as long as the given angle is between the given sides.

(In this diagram we are entering sides 'b' and 'c', and the angle 'A' which is between 'b' and 'c'.)

<u>To enter a value</u>, click on its edit box, then type the value.



IMPORTANT
Do not use the degree operator ° for angles, eg type 35 degrees as just 35, <u>not</u> as 35°.

Click the 'Apply' button at the bottom of the dialog box. The calculated values will display on the options box.

Click the 'OK' button to close the options box. The calculations and triangle diagram will be displayed on your screen.

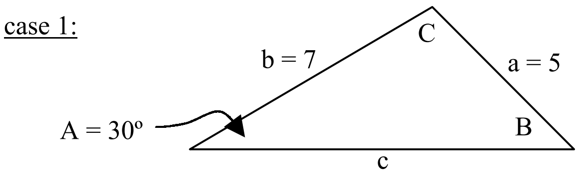
NOTE: **If the diagram becomes too big** for your computer screen, press the 'F10' key to make it smaller.
To make the diagram bigger, hold down 'Shift' while you press 'F10'.

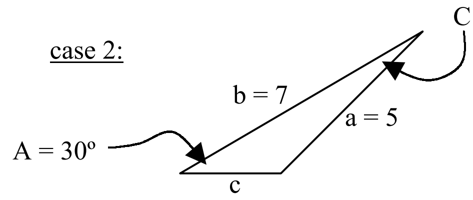
Name:___________________________________ Class:__________

Trigonometry - Activity 23

General Triangle Solution: Given two sides and a non-included angle.

A 'non-included' angle is an internal angle of a triangle that does not lie between two given sides. If two sides and a non-included angle are given for a triangle, then there may be up to two different possible triangles that have these measurements. For example, both triangles in the diagram below have sides 7 and 5, and a non-included angle of 30°...





We will demonstrate the procedure for solving a triangle given two sides and a non-included angle, using the triangle given above. These are the steps:

1. Use the sine rule to find the unknown angle that is opposite one of the given sides.

Subtract this angle from 180º to find a second angle.

Test your calculated angles by comparing them with the given angle. In this case, because side b > side a, angle B must be > angle A. Since both values of B we have found are greater than A = 30º, we can accept them both. If either B value were less than 30º it would mean that this triangle only had one solution.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\sin B = \frac{b \sin A}{a}$$

$$= \frac{7 \sin 30°}{5}$$

$$= 0.7$$

Case 1: Find the inverse sin of 0.730588 using a scientific calculator...

$$B = \sin^{-1}(0.7) = 44.427º$$

Subtract this angle from 180º to find a second value for angle A:

Case 2:

$$B = (180º - 44.427º) = 135.573º$$

2. Use the 'sum of internal angles' rule to find the remaining angle.

The sum of the internal angles equals 180º ...

$$A + B + C = 180º$$

so

Case 1:

C = 180º – (A+B)
 = 180º – (30º + 44.427º)
 = 180º – 74.427º
 = 105.573º

Case 2:

C = 180º – (A+B)
 = 180º – (30º + 135.573º)
 = 180º – 165.573º
 = 14.427º

3. Use the sine rule to find the remaining unknown side.

Case 1:

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$c = \frac{a \sin C}{\sin A}$$

$$= \frac{5 \sin 105.573°}{\sin 30°}$$

$$= 9.63289$$

Case 2:

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

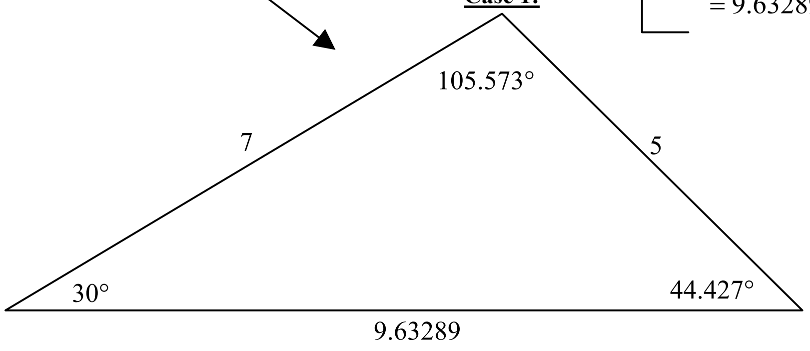
$$c = \frac{a \sin C}{\sin A}$$

$$= \frac{5 \sin 14.427°}{\sin 30°}$$

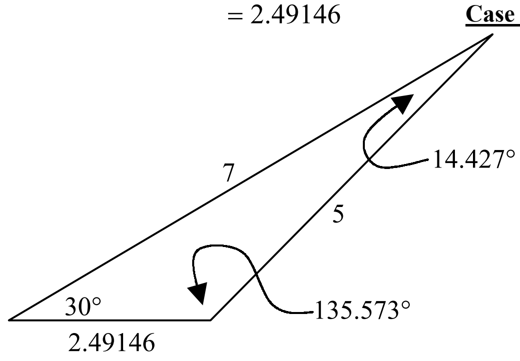
$$= 2.49146$$

These triangles show the case 1 and case 2 solutions...

Case 1:



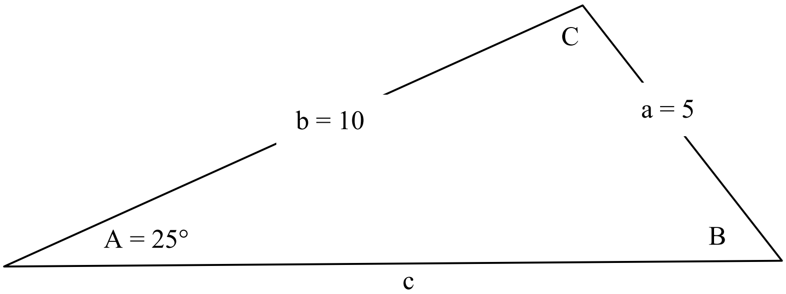
Case 2:

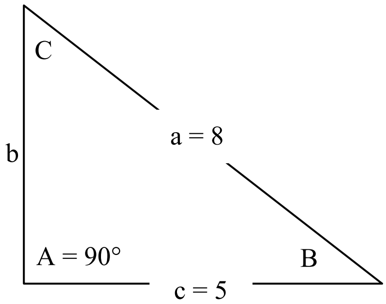


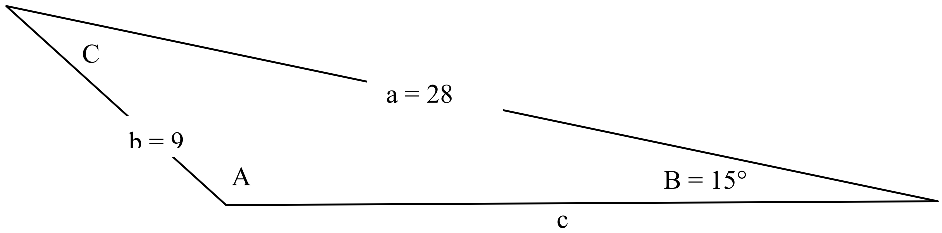
1) In step 1 of the triangle solution on the front of this sheet, a scientific calculator is used to find the inverse sine of 0.7. This is angle B, and = 44.427º.
Explain why another value of angle B is given by : (180º - 44.427º) = 135.573º

2) Given two sides and a non-included angle, sometimes there are two possible triangles as in the example on the front of this sheet, but sometimes there may be only 1, or even none at all.
Explain what will happen in step 1 of the procedure for each of these situations.

3) Three triangles are shown below. In each case, two sides and a non-included angle are given. For each triangle:
(a) If possible, sketch on the diagrams to show how a second triangle can be constructed with the same three measurements. Shade the second triangle (if there is one).
(b) Solve the triangle for the unknown side and angles using the method on the front of this sheet.
Draw labelled diagrams of the triangles.





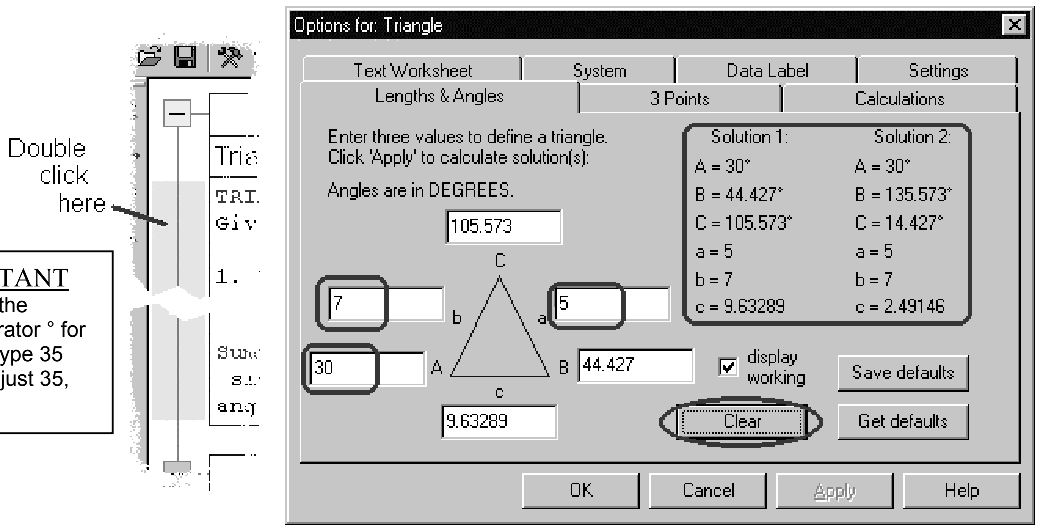


To correct your work...

2) Start Maths Helper Plus and load the ‘Triangle_sover_ASS.mhp’ document.
This document shows the working steps and a diagram for solving triangles when given two sides and a non-included angle.

3) Display the triangle solver options box
Double click the mouse in the border to the left of the calculations. (This area is shaded in the diagram below.)
The triangle solver options box will display its 'Lengths & Angles' tab...

IMPORTANT
Do not use the degree operator ° for angles, eg type 35 degrees as just 35, not as 35°.



Click the 'Clear' button to remove the previous triangle. Enter the sides and angles into three of the white edit boxes.

Click the 'Apply' button at the bottom of the edit box. The calculated values will display on the options box.

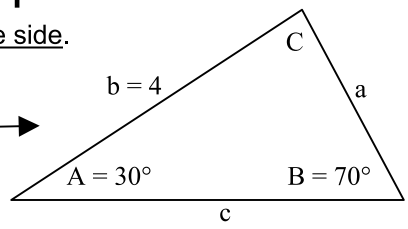
Click the 'OK' button to close the options box. The calculations and triangle diagram will be displayed on your screen.

NOTE: **If the diagram becomes too big** for your computer screen, press the ‘F10’ key to make it smaller.
To make the diagram bigger, hold down ‘Shift’ while you press ‘F10’.

Trigonometry - Activity 24

General Triangle Solution: Given two angles and one side.

When two internal angles and one side of a triangle are known, the other two internal angle and the unknown sides can be found.



These are the formulas used to solve this type of problem:

The **sum of the internal angles** equals 180º ...

$$A + B + C = 180^\circ$$

The '**sine rule**' ...

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

We will demonstrate the procedure for solving a triangle given two angles and one side, using the triangle given above. These are the steps:

1. Use the 'sum of internal angles' rule to find the unknown angle.

The sum of the internal angles equals 180º ...

$$A + B + C = 180^\circ$$

so

$$\begin{aligned} C &= 180^\circ - (A+B) \\ &= 180^\circ - (30^\circ + 70^\circ) \\ &= 180^\circ - 100^\circ \\ &= 80^\circ \end{aligned}$$

2. Use the sine rule to find either of the two unknown sides.

Finding side 'c':

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\begin{aligned} c &= \frac{b \sin C}{\sin B} \\ &= \frac{4 \sin 80^\circ}{\sin 70^\circ} \\ &= 4.19204 \end{aligned}$$

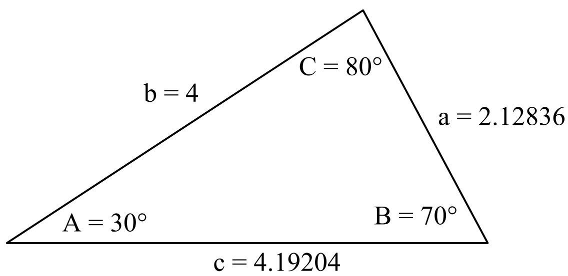
3. Use the sine rule again to find the remaining unknown side.

Finding side 'a':

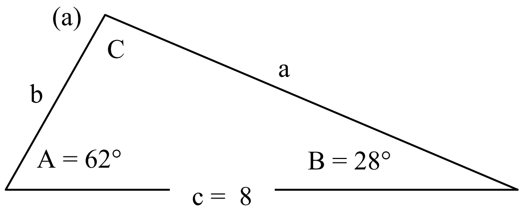
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

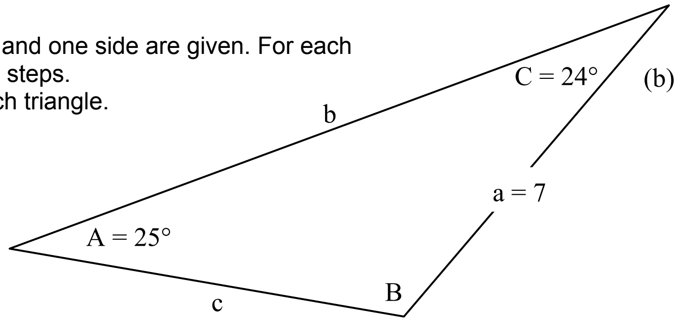
$$\begin{aligned} a &= \frac{b \sin A}{\sin B} \\ &= \frac{4 \sin 30^\circ}{\sin 70^\circ} \\ &= 2.12836 \end{aligned}$$

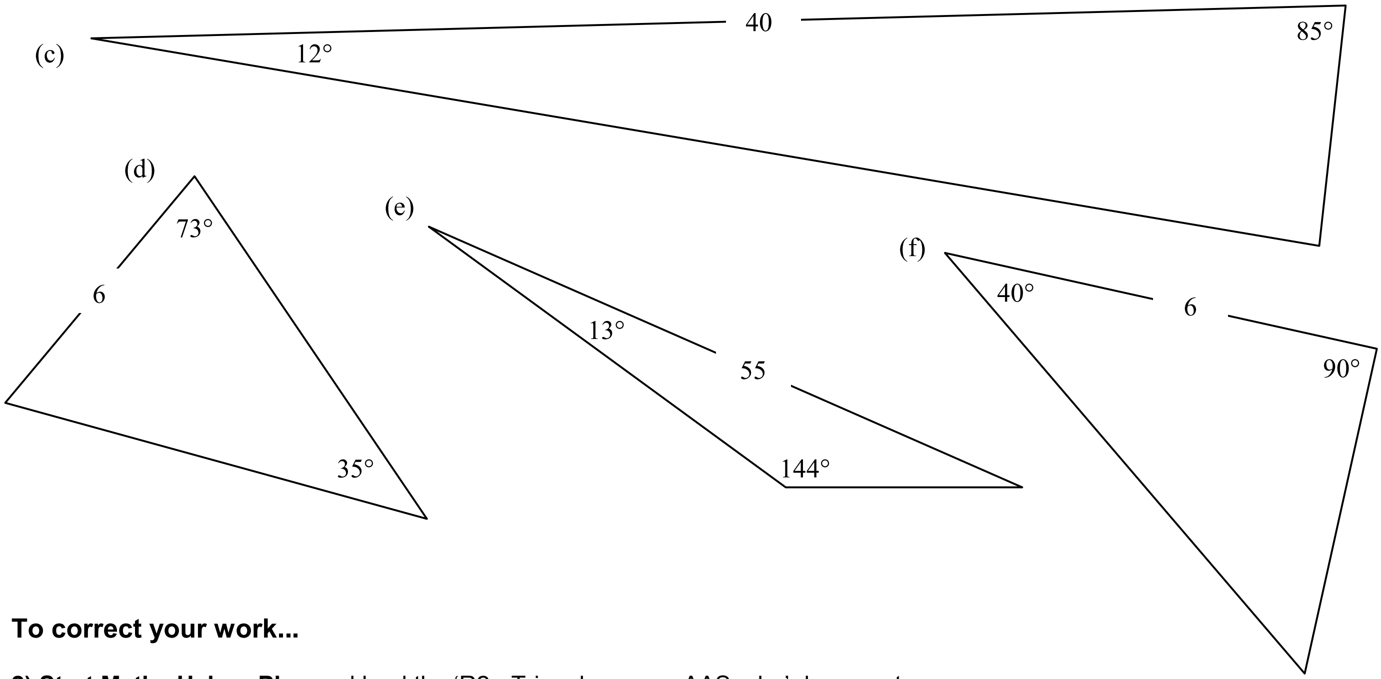
The triangle is now solved. This diagram shows all of the sides and angles:



1) Six triangles are shown below. In each case, two angles and one side are given. For each triangle, find the unknown angle and sides. Show all working steps.
Use Maths Helper Plus to correct your work after solving each triangle.



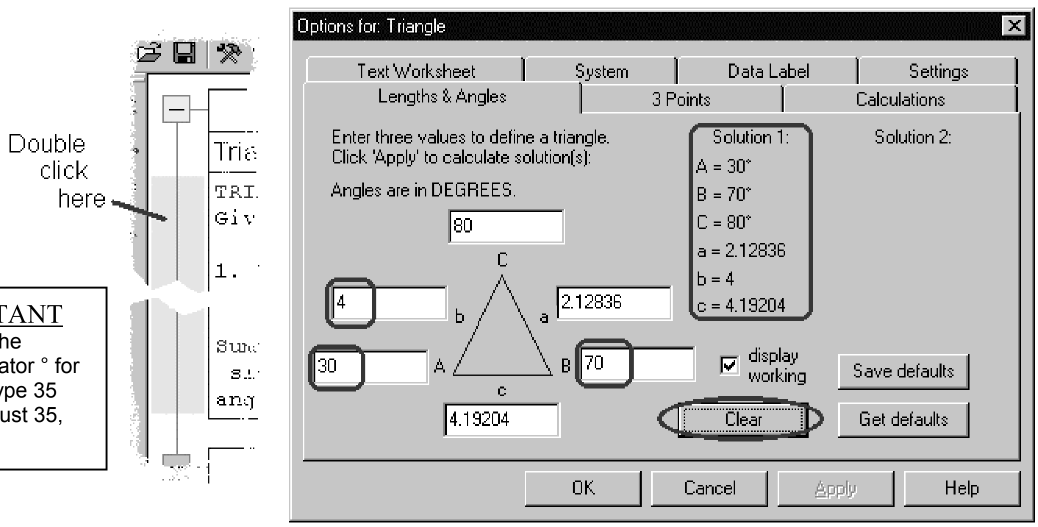




To correct your work...

2) Start Maths Helper Plus and load the 'R2 - Triangle_sover_AAS.mhp' document.
This document shows the working steps and a diagram for solving triangles when given two angles and one side.

3) Display the triangle solver options box
Double click the mouse in the border to the left of the calculations. (This area is shaded in the diagram below.)
The triangle solver options box will display its 'Lengths & Angles' tab...



IMPORTANT
Do not use the degree operator ° for angles, eg type 35 degrees as just 35, not as 35°.

Click the 'Clear' button to remove the previous triangle, then enter the two known angles and side length into the white edit boxes.

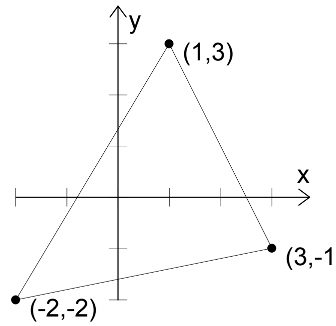
Click the 'Apply' button at the bottom of the edit box. The calculated values will display on the options box.

Click the 'OK' button to close the options box. The calculations and triangle diagram will be displayed on your screen.

NOTE: **If the diagram becomes too big** for your computer screen, press the 'F10' key to make it smaller.
To make the diagram bigger, hold down 'Shift' while you press 'F10'.

Trigonometry - Activity 25

General Triangle Solution: Given three (x,y) points.



Plot three points on the (x,y) plane and join them with lines. If the points are not in the same straight line, you will have created a triangle. This diagram shows the triangle created by the three points (1,3), (-2,-2) and (3,-1):

The distance formula can be used to find the distance between two (x,y) points. For example, consider the points (x_1, y_1) and (x_2, y_2). The distance formula gives the distance 'd' between them as follows:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

We can use the distance formula to find the distance between each pair of points making up our triangle. These distances are the lengths of the three sides.

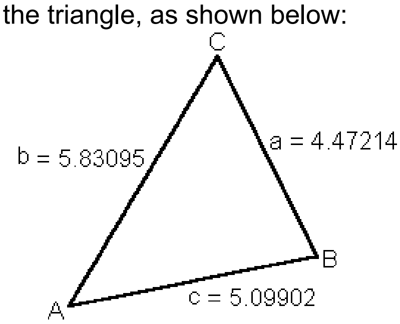
For the example above, the points are: (1,3), (-2,-2) and (3,-1), so we calculate the side lengths by taking these points two at a time:

For the side joining (1,3) and (-2,-2) we have:

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-2-1)^2 + (-2-3)^2} \\ &= \sqrt{34} \\ &= 5.83095 \end{aligned}$$

Similarly, the side joining (1,3) and (3,-1) is 4.47214, and the side joining (-2,-2) and (3,-1)is 5.09902

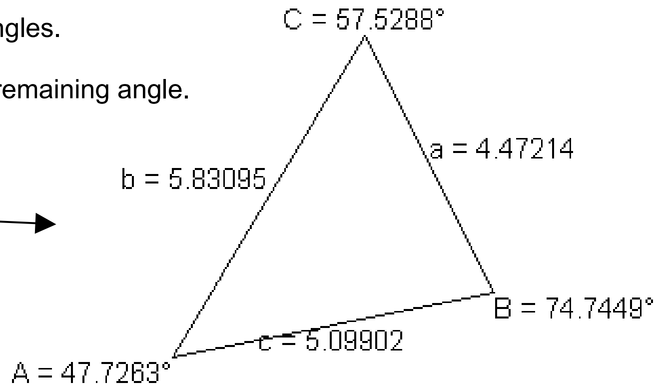
We now know the three side lengths of the triangle, as shown below:



Once the three side measurements are known, then the internal angles 'A', 'B' and 'C' can be found using the procedure for a triangle when given the three side lengths. In summary, this method is as follows:

1. Use the cosine rule to find the largest angle (which is opposite the largest side).
2. Use the sine rule to find one of the remaining angles.
3. Use the 'sum of internal angles' rule to find the remaining angle.

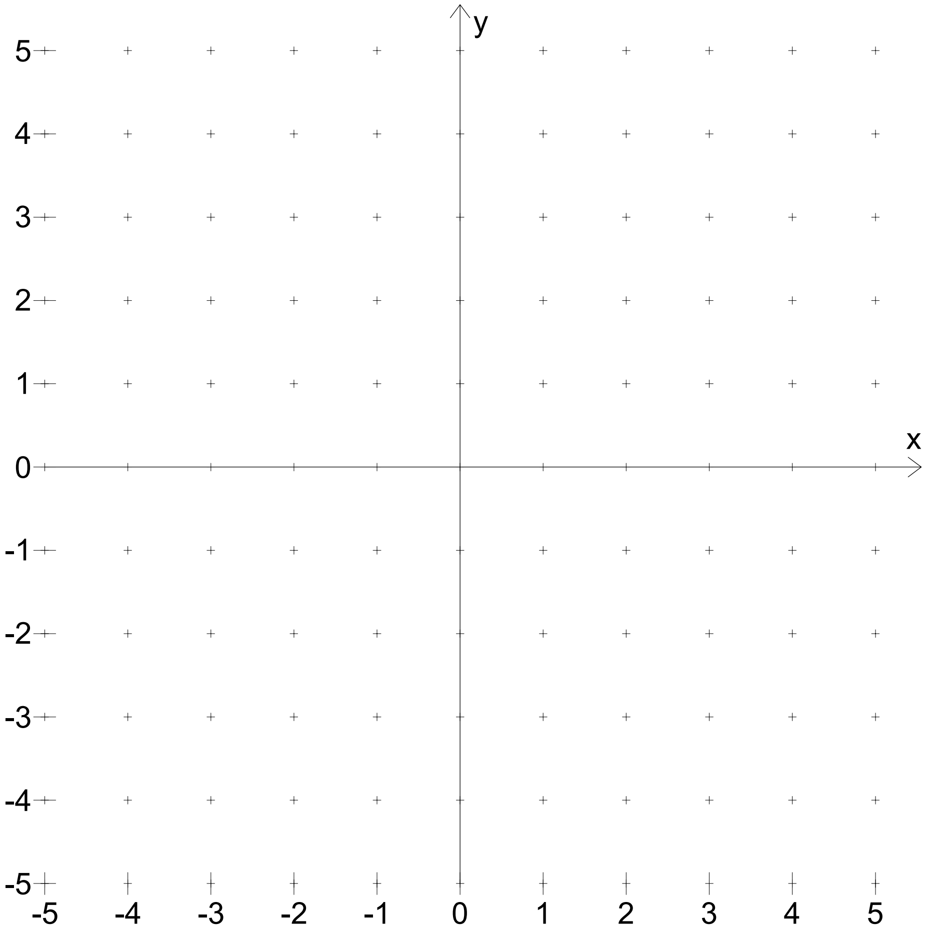
For our example, the final result is as follows:



1) Six triangles are defined by the (x,y) points given below. Plot each set of points on the graph grid below, then find the unknown angle and sides. Show all working steps. Use Maths Helper Plus to correct your work after solving each triangle.

(a) (0,2) (2,5) (5,2)   (b) (-2,-3) (-2,-5) (5,-3)   (c) (1,5) (-5,4) (-2,2)

(d) (1,-1) (-5,-3) (5,-2)   (e) (-5,3) (-4,-2) (-1,1)   (f) (5,0) (-1,2) (2,-1)

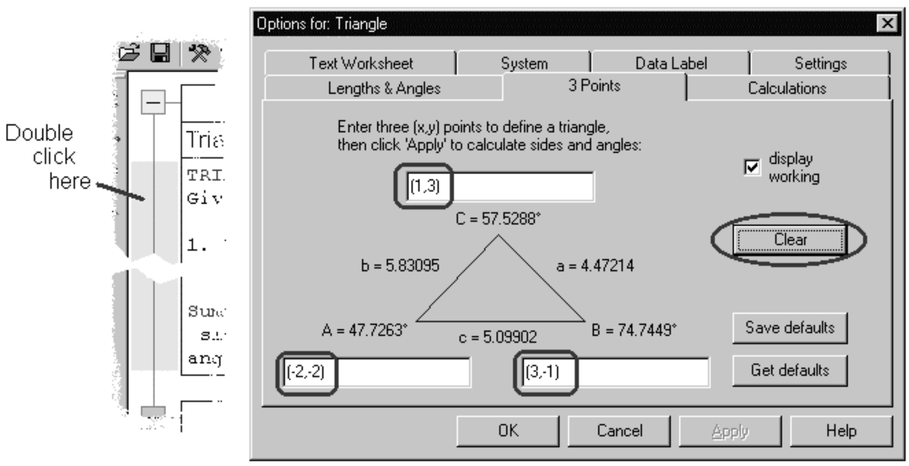


To correct your work...

2) Start Maths Helper Plus and load the 'R2 - Triangle_sover_3_points.mhp' document.
This document shows the working steps and a diagram for solving triangles when given three (x,y) points.

3) Display the triangle solver options box

Double click the mouse in the border to the left of the calculations. (This area is shaded in the diagram below.)
The triangle solver options box will display its '3 Points' tab...



Click the 'Clear' button to remove the previous triangle, then enter the three (x,y) points into the white edit boxes.

Click the 'Apply' button at the bottom of the edit box. The calculations will appear on the text view, and the diagram on the graph view of Maths Helper Plus.

Click the 'OK' button to close the options box and view the calculations and diagram.

NOTE: **If the diagram becomes too big** for your computer screen, press the 'F10' key to make it smaller.
To make the diagram bigger, hold down 'Shift' while you press 'F10'.