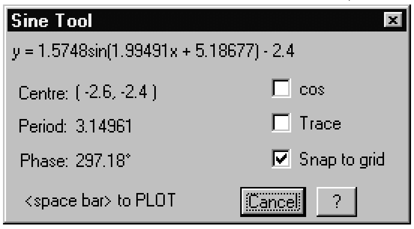
Trig Functions - Activity 1

An investigation of amplitude and period of sine functions.

1) Start Maths Helper Plus, or create a new document. Use the 'Use template...' command in the 'File' menu to load the template file: 'Graph setup - trig 1.tpl'

2) Select the 'Sine' command from the 'Tools' menu. The sine tool dialog box will appear on the screen: Drag on its title bar to move it to a convenient location so that you can see all of the graphing area. Make sure only the 'Snap to grid' option is selected:

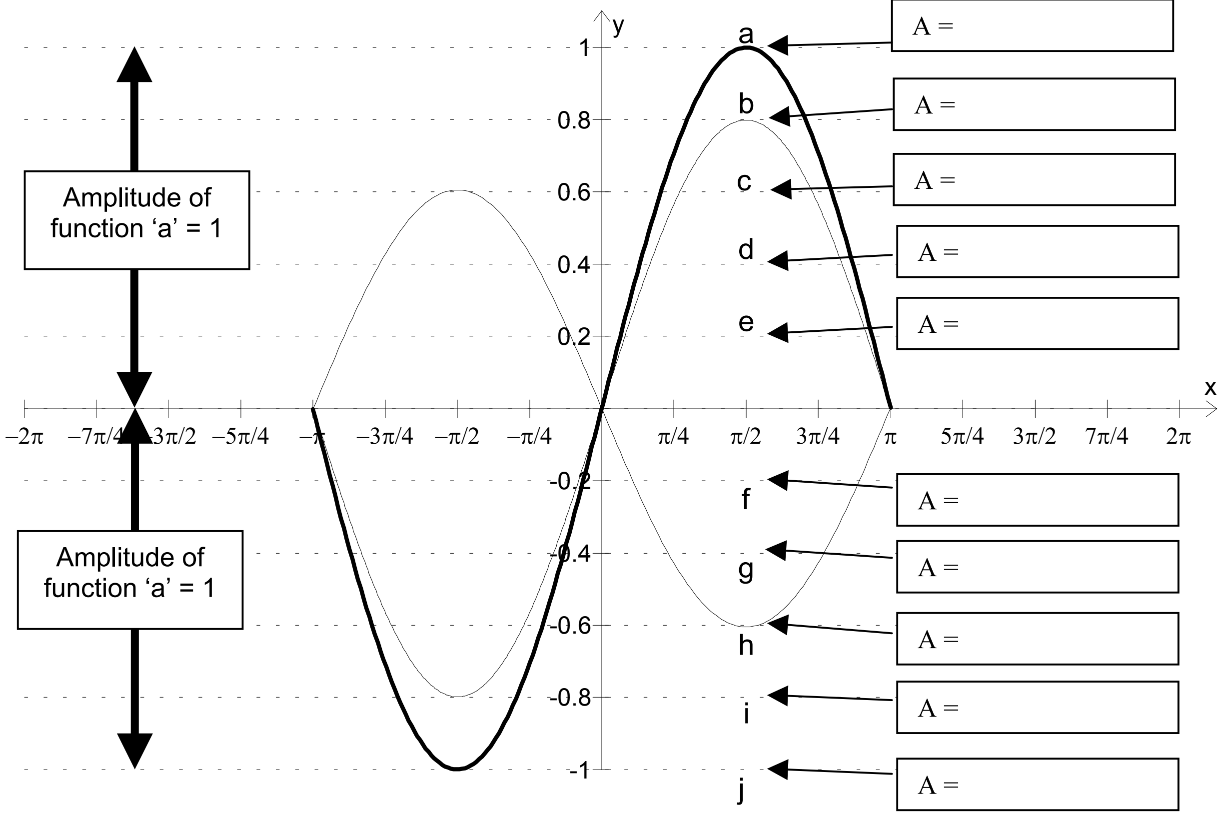


3) Move the mouse cursor over the graph view, and the sine function graph will appear. The equation of this graph is at the top of the sine tool dialog box. It has the form: $y = A\sin(Bx + C) + D$

4) To change the shape of the sine curve, drag with the mouse in any direction. Try dragging the mouse from side to side, then up and down. (Press 'Shift' while you drag for finer control.)

5) If 'C' and 'D' are zero, then the sine function is: $y = A\sin(Bx)$. Move the sine tool cursor so that it is centred on the origin. The sine equation will now have the form: $y = A\sin(Bx)$.

By dragging the mouse, <u>change the shape of the sine graph</u> so that it looks like curve 'a' on this diagram. <u>Press the space bar</u> to plot the graph. <u>Write the 'A' value</u> from the sine equation in the box provided on the diagram. <u>Repeat for each of the other graphs</u> 'b' to 'j' on this diagram, writing the 'A' values in the boxes each time.

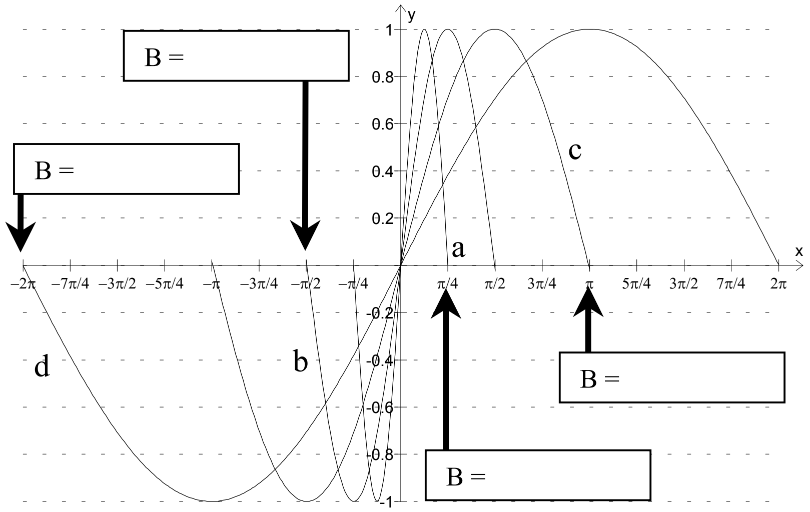


The maximum distance of the highs and lows of a sine function from the mean 'y' value is called the 'amplitude' of the function. (See diagram above.) In this case, the mean 'y' value of the function is zero because it is centred on the 'x' axis, so the amplitude equals the absolute (positive) value of the maximum or minimum 'y' coordinate. For function 'a' plotted above, the amplitude is close to 1.

6) How can the amplitude of a sine function be found from its equation ?

7) What does a negative value of 'A' do to the graph of a sine function ?

8) Repeat step 1 above to create a new document.



9) <u>Position the sine tool cursor</u> in each of the four positions 'a' to 'd' as illustrated in this diagram. In each case, <u>press the space bar</u> to plot the graph, and <u>write the 'B' value</u> from the sine function in the box provided.

Sine functions are 'periodic', which means that the same pattern is repeated over and over. The sine tool normally shows only one cycle of the function, and its width is the period. For example, the repeating pattern for graph 'c' in the diagram begins at x = $-\pi$ and finishes at x = π. This distance is the period, which is 2π. The symbol 'T' is often used for the period.

10) Fill in the 'B' and 'Period, T' columns of the table below using information from question 9.
Then calculate and write the '$(2\pi)/B$' values in the last column.

Graph	B	Period, 'T'	$\frac{2\pi}{B}$
a			
b			
c			
d			

11) From the table above, deduce the relationship between the period, 'T' of a sine function and the 'B' value. Write as an equation:

12) Use what you have learned to make rough sketches of these sine functions:

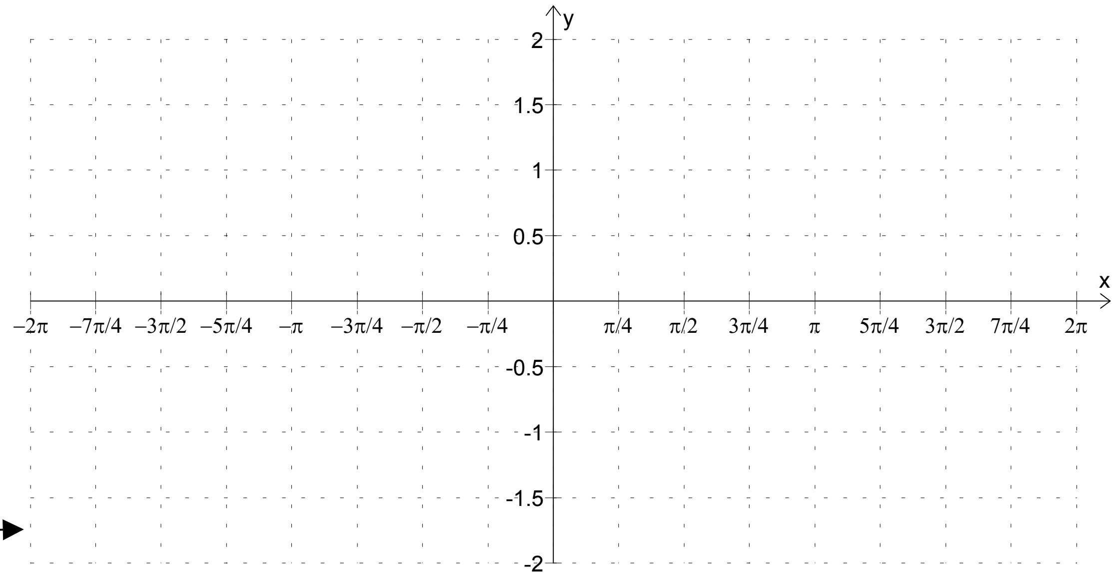
a) y = 2sinx

b) y = -sinx

c) y = 1.5sin(2x)

d) y = -2sin(0.5x)

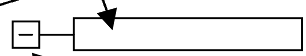
Draw the graphs here:



13) Use Maths Helper Plus to test your answers. Repeat step 1 above to create a fresh graph, but use the template file: 'Graph setup - trig 2.tpl'. Graph each of these equations and compare the graphs with your sketches. Modify any of your sketches that are very different to the correct graphs.

To graph an equation you:

- Click on the input box. (On the text view.)
- Type the equation you want to check.
- Click outside of the input box.

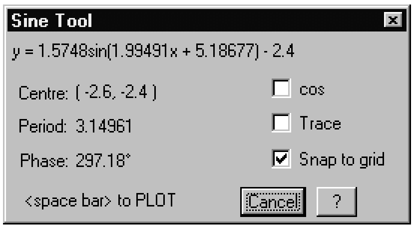


Trig Functions - Activity 2

An investigation of phase and vertical position of sine functions.

1) Start Maths Helper Plus, or create a new document. Use the 'Use template...' command in the 'File' menu to load the template file: 'Graph setup - trig 1.tpl'

2) Select the 'Sine' command from the 'Tools' menu. The sine tool dialog box will appear on the screen:
Drag on its title bar to move it to a convenient location so that you can see all of the graphing area. Make sure only the 'Snap to grid' option is selected:

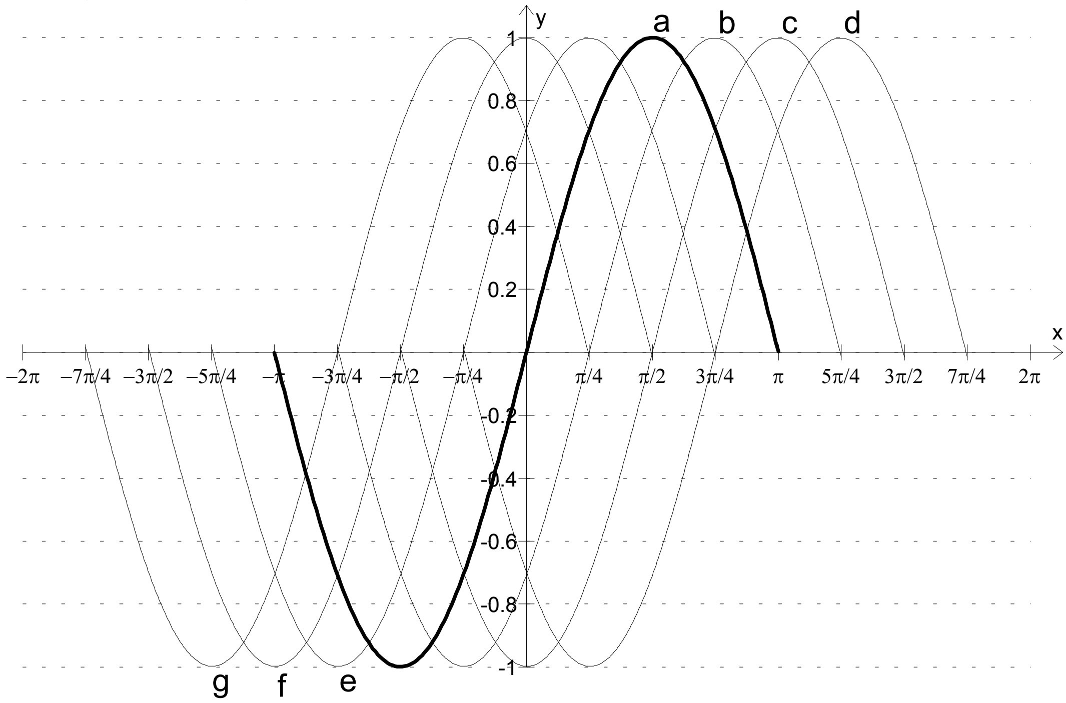


3) Move the mouse cursor over the graph view, and the sine function graph will appear. The equation of this graph is at the top of the sine tool dialog box. It has the form: $y = A\sin(Bx + C) + D$

4) To change the shape of the sine curve, drag with the mouse in any direction. Try dragging the mouse from side to side, then up and down. (Press 'Shift' while you drag for finer control.)

5) Move the sine tool cursor so that it is centred on the origin.
The sine equation will now have the form: $y = A\sin(Bx)$.

By dragging the mouse, <u>change the shape</u> of the sine graph so that it looks like curve 'a' below. <u>Press the space bar</u> to plot the graph. <u>Record</u> the 'B' and 'C' values from the sine equation in the table at the bottom of this page. <u>Without clicking any mouse buttons, move</u> the mouse horizontally to position the sine curve for graphs 'b' to 'g', below. Record the 'B' and 'C' values as before.



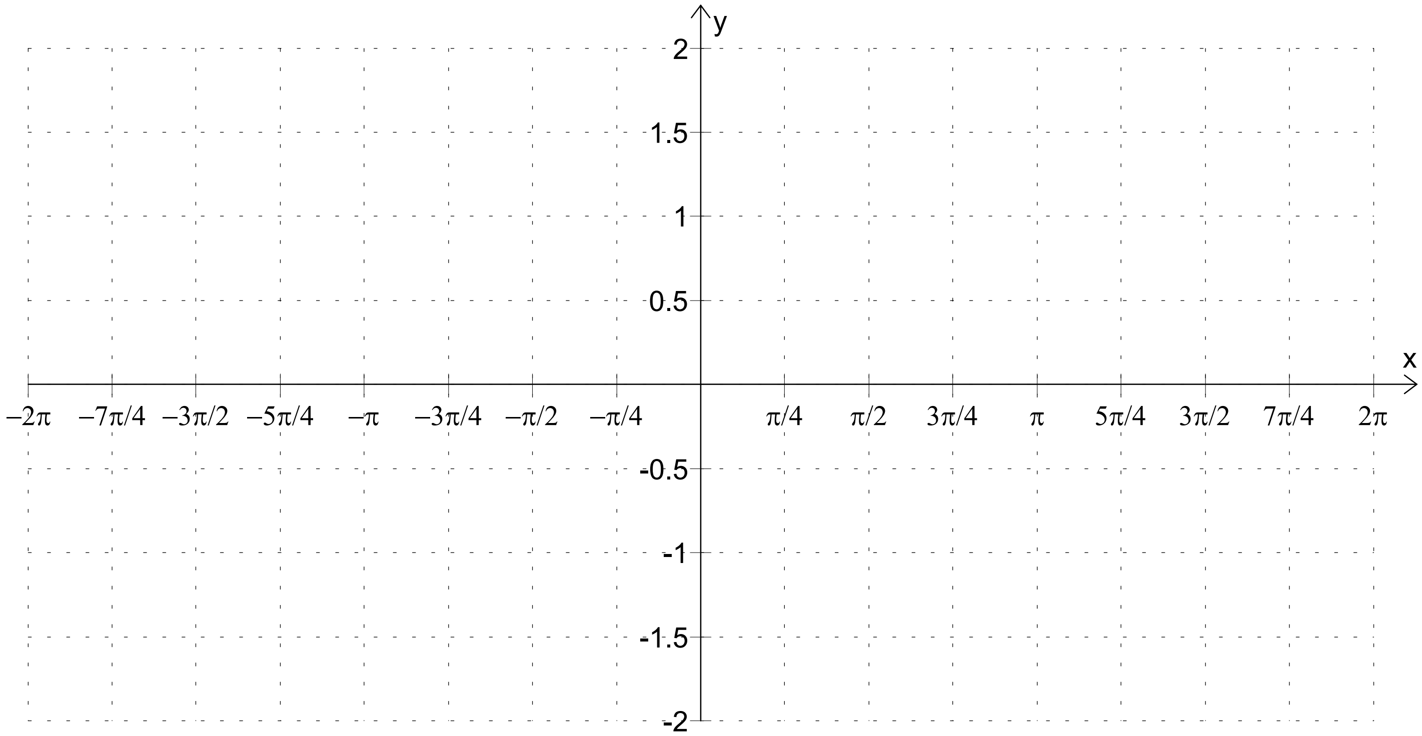
Graph:	B	C	C/B	Phase shift
a				
b				
c				
d				
e				
f				
g				

6) The displacement of a sine graph in the ‘x’ direction is called its ‘phase shift’. Positive phase shifts displace a graph in the negative ‘x’ direction, while negative phase shifts displace it in the positive ‘x’ direction. Thus for the graphs on the front of this sheet, the phase shifts are: 0 for graph ‘a’, $-\pi/4$ for graph ‘b’, and $\pi/2$ for graph ‘f’. Determine the phase shifts for each of the other sine graphs and then complete the last column of the table.

7) From the known values of ‘C’ and ‘B’, calculate C/B for each graph and so complete the third column of the table. Write down an equation that describes the relationship between the quantity: C/B and the phase shift of a sine function.

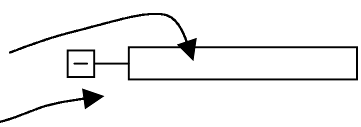
8) Make rough sketches of these sine functions on the graph grid below:

a) $y = \sin(x + \pi)$ b) $y = -\sin(x - \pi/2)$ c) $y = 2\sin(2x - \pi)$ d) $y = -1.5\sin(0.5x + 5\pi/4)$



9) Use Maths Helper Plus to test your answers. Repeat step 1 above to create a fresh graph, but use the template file: ‘Graph setup - trig 2.tpl’. Graph each of these equations and compare the graphs with your sketches. Modify any of your sketches that are very different to the correct graphs.

To graph an equation you:
- Click on the input box. (On the text view.)
- Type the equation you want to check.
- Click outside of the input box.



HINT:
In Maths Helper Plus, type ‘pi’ for ‘π’.

An investigation - The ‘D’ value

10) How do you think the ‘D’ value in the sine function: $y = A\sin(Bx + C) + D$ effects the graph ?

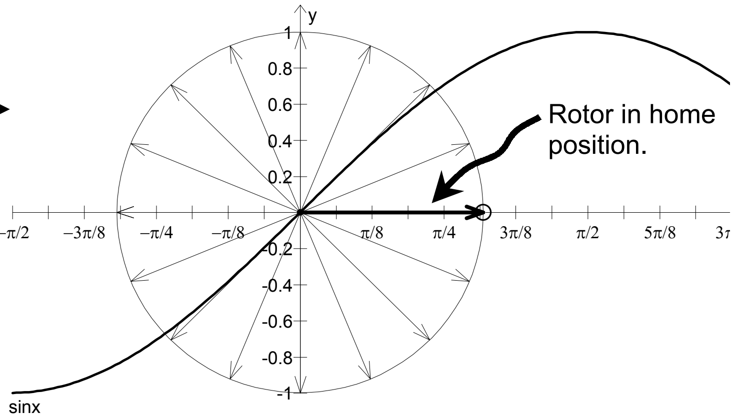
11) Test your theory by experimenting with the sine tool in Maths Helper Plus. Were you right ?

12) On the graph above, sketch graphs of $y = \sin x + 1.5$, and $y = \sin x - 1.5$, then correct your answers by plotting the graphs in Maths Helper Plus.

Trig Functions - Activity 3

An investigation of the sine function with the unit circle.

1) Start Maths Helper Plus, then load the file: 'Trig - Unit circle, Sine.mhp'. Hold down a 'Ctrl' key and press the 'G' key to display only the graph view:



This circle has a radius of 1, and is called a 'unit circle'. The arrows from the centre of the circle divide it into 16 equal arcs, each with central angle = $\pi/8$.

2) Show with calculations that the angle between the arrows is $\pi/8$:

3) What is the length of the arc along the circle from any one arrow point to the next ?_______________

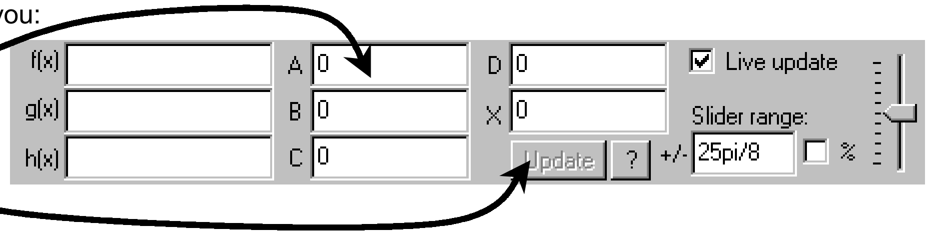
The thick (pink coloured) arrow can be rotated about the centre of the unit circle. Let's call it the 'rotor'. The 'home position' of the rotor is when it is pointing in the positive 'x' direction. (See diagram above.)

As the rotor rotates around the unit circle, it sweeps out angle 'A', measured in radians. Positive angles are measured anticlockwise from the home position, and negative angles clockwise. You use the 'parameters box' in Maths Helper Plus to change the value of 'A' to rotate the rotor.

4) Press the F5 key to display the parameters box. (See below).

To set the 'A' to a value immediately you:

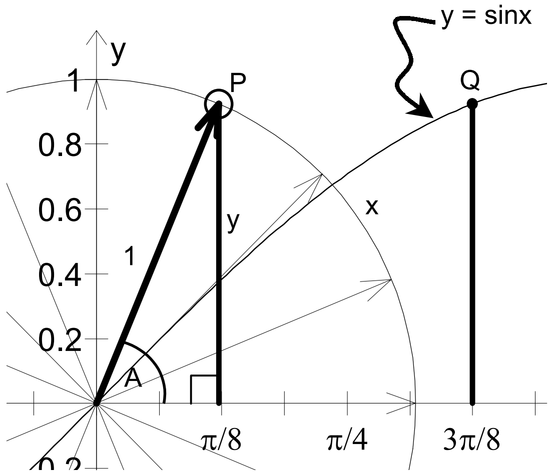
1) Click on the 'A' edit box,
2) Type the value for 'A',
3) Click the 'Update' button.



5) Set parameter 'A' to $3\pi/8$ (you type: 3pi/8). The rotor will move to its new position. (See below)

The rotor (length = 1) is now the hypotenuse of a right triangle with angle 'A' at the origin.

If 'P' is where the rotor meets the unit circle, show that the 'y' coordinate of 'P' is given by sinA.



6) How far has 'P' moved along the circle as 'A' has changed from zero to $3\pi/8$?

The 'y' coordinate of 'P' is the length of the vertical side of the triangle (green coloured on the computer screen).

We can now define the 'sine' function: y = sinx in terms of the unit circle:

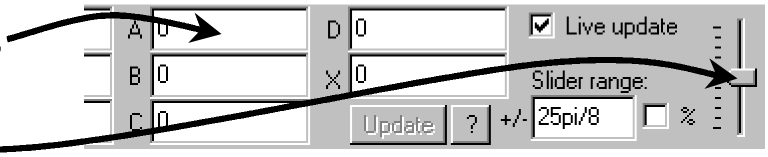
- 'x' is the arc length from the home position to 'P', and so 'x' can be any real number, and
- 'y' is the 'y' coordinate of the rotor.

Point 'Q' on the diagram lies on the graph of y = sinx. Its 'y' coordinate is always the same as that of the rotor, and its 'x' coordinate is the arc length around the unit circle from the home position to 'P'.
The function y = sinx is made up of all possible values of 'Q'. This function has been plotted in blue.

You will now use the unit circle to explore some of the properties of the function: y = sinx.

To do this, you need to be able to change the 'A' value more easily. This is how to change 'A' using the arrow keys on your computer keyboard:

a) (FIRST TIME ONLY) Click on the 'A' edit box,
b) (OPTIONAL) Type the initial value for 'A'
c) Click the slider button.



Now you can use the up and down arrow keys to change 'A'.
Hold down an arrow key to create a smooth animation effect.
(If the arrow keys stop responding, click the slider button again.)

7) Click the '—' button on the standard toolbar until you can see all of the plotted sine graph.
Set 'A' to 0, click the slider button again, then use the arrow keys on the keyboard to change the 'A' value smoothly. Watch the rotor and point 'Q' on the sine function graph as 'A' changes for both positive and negative 'A' values.

Return the 'A' value to zero, and click the '+' button on the standard to restore the graph to its original size.

By changing 'A', answer these questions:

8) What are the maximum and minimum values of the function y = sinx ? ____________ ____________

9) A periodic function consists of a repeating pattern. The width of the pattern is called the 'period' of the function. By inspecting the graph, determine the period of the function: y = sinx. _______________

10) Inspect the graph of y = sinx, then write two 'x' values that satisfy each of these equations:

a) sin(x) = 1 _______ _______ sin(x) = 0 _______ _______ sin(x) = -1 _______ _______

11) Use the arrow keys (as described above) to change 'A' to each of these:
$\pi/8$ and $7\pi/8$, $\pi/4$ and $3\pi/4$, $3\pi/8$ and $5\pi/8$.
Compare the value of sinx for each pair of 'A' values. What did you find ?

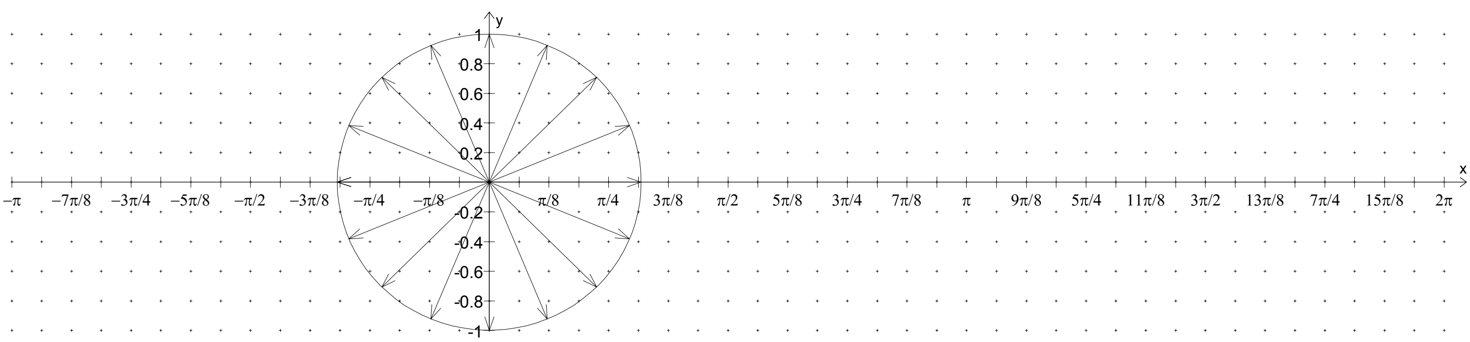
12) From your observations in question (11), complete this equation: $\sin(\pi-x)$ = ____________________

13) How many values of 'x' satisfy the equation: sinx = $\sin(\pi/8)$? ____________________
(Hint: The function: y = sinx goes to infinity in both directions.)

14) Make observations with the unit circle (similar to question 11 above), then complete these equations:

$\sin(\pi+x)$ = ____________________ $\sin(-x)$ = ____________________

15) Sketch a graph of the function: y = sin(x) on the grid below. First mark the points where the function cuts the 'x' axis, as well as maximum and minimum points. Use the unit circle on this diagram to help locate some intermediate points. Finally, join the points with smooth lines.

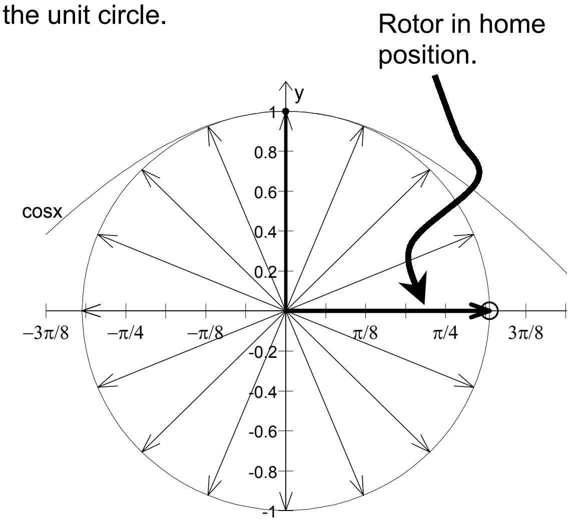


Name:______________________________________ Class: __________

Trig Functions - Activity 4

An investigation of the cosine function with the unit circle.

1) Start Maths Helper Plus, then load the file: 'Trig - Unit circle, Cos.mhp'. Hold down a 'Ctrl' key and press the 'G' key to display only the graph view:



This circle has a radius of 1, and is called a 'unit circle'. The arrows from the centre of the circle divide it into 16 equal arcs, each with central angle = $\pi/8$.

2) Show with calculations that the angle between the arrows is $\pi/8$:

3) What is the length of the arc along the circle from any one arrow point to the next ?

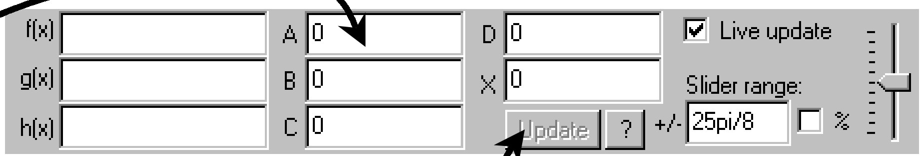
The thick (pink coloured) arrow can be rotated about the centre of the unit circle. Let's call it the 'rotor'. The 'home position' of the rotor is when it is pointing in the positive 'x' direction. (See diagram above.)

As the rotor rotates around the unit circle, it sweeps out angle 'A', measured in radians. Positive angles are measured anticlockwise from the home position, and negative angles clockwise. You use the parameters box in Maths Helper Plus to change the value of 'A' to rotate the rotor.

4) Press the F5 key to display the parameters box. (See below).

To set the 'A' to a value immediately you:

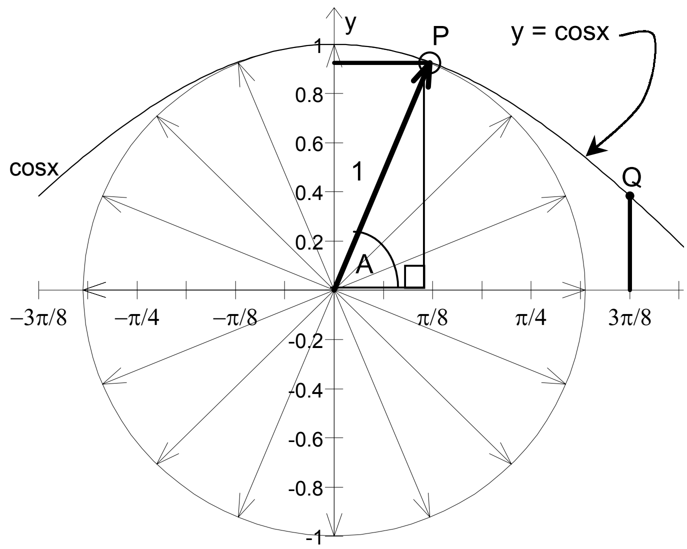
1) Click on the 'A' edit box,
2) Type the value for 'A',
3) Click the 'Update' button.



5) Set parameter 'A' to $3\pi/8$ (you type: 3pi/8). The rotor will move to its new position. (See below)

The rotor (length = 1) is now the hypotenuse of a right triangle with angle 'A' at the origin.

If 'P' is where the rotor meets the unit circle, show that the 'x' coordinate of 'P' is given by cosA.



6) How far has 'P' moved along the circle as 'A' has changed from zero to $3\pi/8$?

The 'x' coordinate of 'P' is the length of the horizontal side of the triangle (green coloured on the computer screen).

We can now define the 'cosine' function: y = cosx in terms of the unit circle:

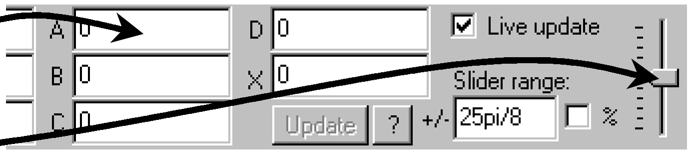
- 'x' is the arc length from the home position to 'P', and so 'x' can be any real number, and
- 'y' is the 'x' coordinate of 'P'.

Point 'Q' on the diagram lies on the graph of y = cosx. Its 'y' coordinate is always the same as the 'x' coordinate of point 'P', and its 'x' coordinate is the arc length around the unit circle from the home position to 'P'.
The function y = cosx is made up of all possible values of 'Q'. This function has been plotted in blue.

You will now use the unit circle to explore some of the properties of the function: y = cosx.

To do this, you need to be able to change the 'A' value more easily. This is how to change 'A' using the arrow keys on your computer keyboard:

a) (FIRST TIME ONLY) Click on the 'A' edit box,
b) (OPTIONAL) Type the initial value for 'A'
c) Click the slider button.



Now you can use the up and down arrow keys to change 'A'. Hold down an arrow key to create a smooth animation effect. (If the arrow keys stop responding, click the slider button again.)

7) Click the '–' button on the standard toolbar until you can see all of the plotted cosine graph.
Set 'A' to 0, then use the arrow keys on the keyboard to change the 'A' value smoothly. Watch the rotor and point 'Q' on the cosine function graph as 'A' changes for both positive and negative 'A' values.

Return the 'A' value to zero, and click the '+' button on the standard to restore the graph to its original size.

By changing 'A', answer these questions:

8) What are the maximum and minimum values of the function y = cosx ? ____________ ____________

9) A periodic function consists of a repeating pattern. The width of the pattern is called the 'period' of the function. By inspecting the graph, determine the period of the function: y = cosx. ________________

10) Inspect the graph of y = cosx, then write two 'x' values that satisfy each of these equations:

a) cos(x) = 1 ________ ________ cos(x) = 0 ________ ________ cos(x) = –1 ________ ________

11) Use the arrow keys (as described above) to change 'A' to each of these:

$\pi/8$ and $-\pi/8$, $\pi/4$ and $-\pi/4$, $3\pi/8$ and $-3\pi/8$.

Compare the value of cosx for each pair of 'A' values by observing the position of point 'Q'. What did you find ?

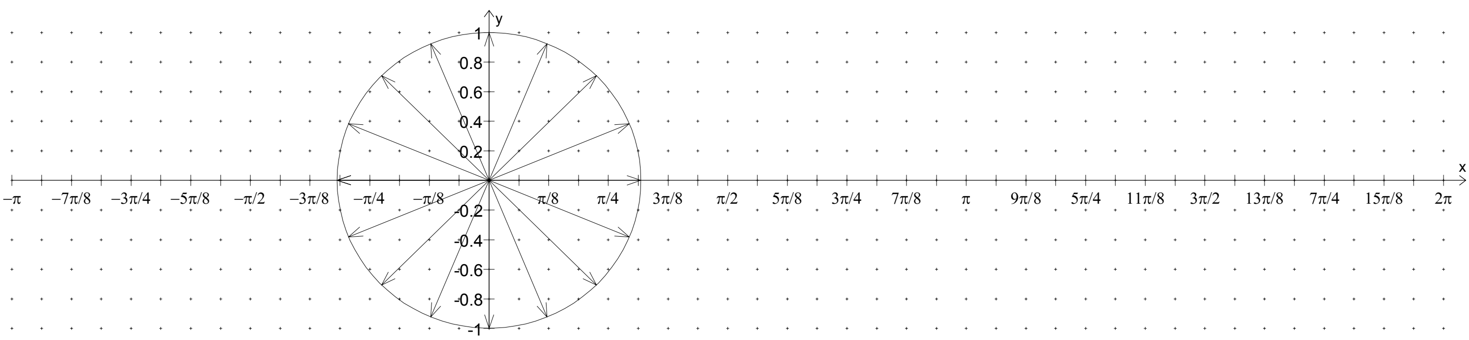
12) From your observations in question (11), complete this equation: cos(–x) = ______________________

13) How many values of 'x' satisfy the equation: cosx = cos(π/8) ? ______________________
(Hint: The function: y = cosx goes to infinity in both directions.)

14) Make observations with the unit circle, then complete these equations:

cos(π+x) = ______________________ cos(π–x) = ______________________

15) Sketch a graph of the function: y = cos(x) on the grid below. First mark the points where the function cuts the 'x' axis, as well as maximum and minimum points. Use the unit circle on this diagram to help locate some intermediate points. Finally, join the points with smooth lines.



Trig Functions - Activity 5

An investigation of the tangent function with the unit circle.

1) Start Maths Helper Plus, then load the file: 'Trig - Unit circle, Tan.mhp'. Hold down a 'Ctrl' key and press the 'G' key to display only the graph view:

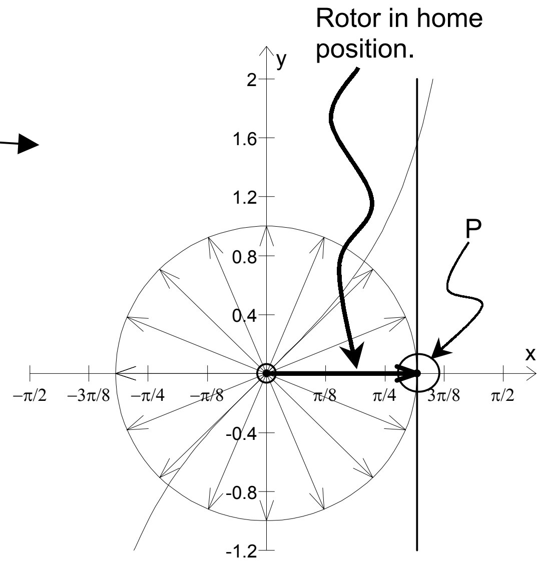


Figure 1

This circle has a radius of 1, and is called a 'unit circle'. The arrows from the centre of the circle divide it into 16 equal arcs, each with central angle = $\pi/8$.

2) Show with calculations that the angle between the arrows is $\pi/8$:

3) What is the length of the arc along the circle from any one arrow point to the next ?

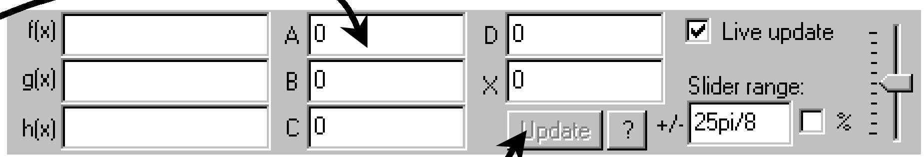
The thick (dark blue coloured) arrow can be rotated about the centre of the unit circle. Let's call it the 'rotor'. The 'home position' of the rotor is when it is pointing in the positive 'x' direction. (See figure 1.)

As the rotor rotates around the unit circle, it sweeps out angle 'A', measured in radians. Positive angles are measured anticlockwise from the home position, and negative angles clockwise. You use the parameters box in Maths Helper Plus to change the value of 'A' to rotate the rotor.

4) Press the F5 key to display the parameters box. (See below).

To set the 'A' to a value immediately you:

1) Click on the 'A' edit box,
2) Type the value for 'A',
3) Click the 'Update' button.



5) Set parameter 'A' to $\pi/8$ (you type: pi/8). The rotor will move to its new position. (See below)

6) If 'P' is where the rotor touches the unit circle, (See magnified circle in figure 2) how far has 'P' moved along the circle as 'A' has changed from zero to $\pi/8$?

7) If the coordinates of 'P' in figure 2 are (a,b), show that $\tan A = b/a$.

8) If the rotor line is extended to meet the line x=1 at 'T', show that the height of 'T' above the 'x' axis = tanA. (Hint, the radius of the unit circle = 1).

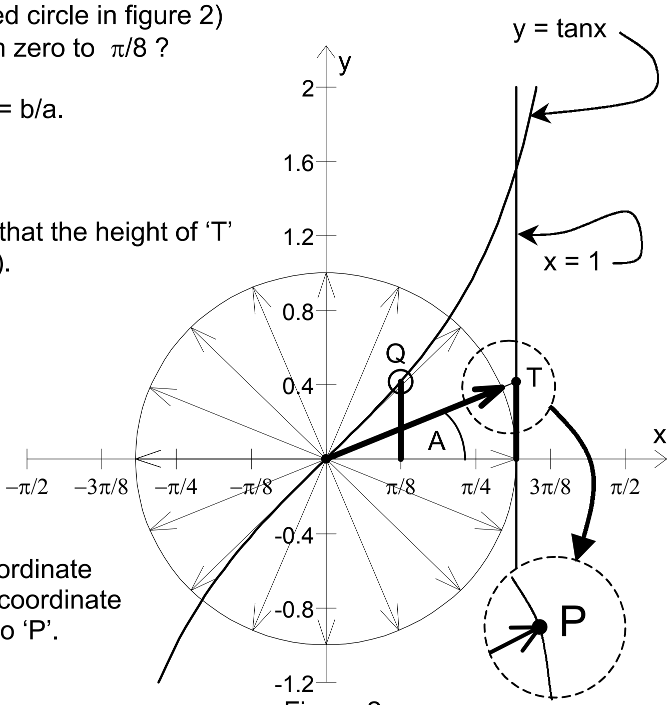


Figure 2

We can now define the 'tangent' function: y = tanx in terms of the unit circle:

- 'x' is the arc length from the home position to 'P', and so 'x' can be any real number, and
- 'y' is the y coordinate of 'T' on the line x=1.

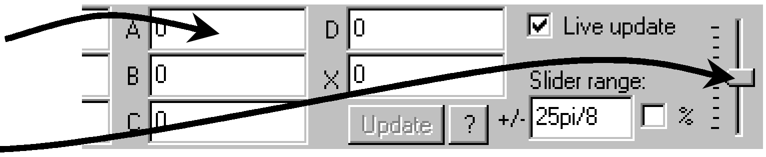
Point 'Q' on the diagram lies on the graph of y = tanx. Its 'y' coordinate is always the same as the 'y' coordinate of point 'T', and its 'x' coordinate is the arc length around the unit circle from the home position to 'P'.

The function y = tanx is made up of all possible values of 'Q'. (This function has been plotted in blue.)

You will now use the unit circle to explore some of the properties of the function: y = tanx.

To do this, you need to be able to change the ‘A’ value more easily. This is how to change ‘A’ using the arrow keys on your computer keyboard:

a) (FIRST TIME ONLY) Click on the ‘A’ edit box,
b) (OPTIONAL) Type the initial value for ‘A’
c) Click the slider button.



Now you can use the up and down arrow keys to change ‘A’. Hold down an arrow key, or press it repeatedly to create a smooth animation effect. (If the arrow keys stop responding, click the slider button again.)

7) Click the ‘—‘ button on the standard toolbar until you can see all of the plotted tangent graph.
Set ‘A’ to 0, then use the arrow keys on the keyboard to change the ‘A’ value smoothly. Watch the point ‘T’ on the line x = 1, and point ‘Q’ on the tangent function graph as ‘A’ changes for both positive and negative ‘A’ values.

Return the ‘A’ value to zero, and click the ‘+’ button on the standard to restore the graph to its original size.

By changing ‘A’, answer these questions:

8) What are the maximum and minimum values of the function y = tanx ? ____________ ____________

9) A periodic function consists of a repeating pattern. The width of the pattern is called the ‘period’ of the function. By inspecting the graph, determine the period of the function: y = tanx.

10) Use the arrow keys (as described above) to change ‘A’ to each of these:

$\pi/8$ and $-\pi/8$, $\pi/4$ and $-\pi/4$, $3\pi/8$ and $-3\pi/8$.

Compare the value of tanx for each pair of ‘A’ values by observing the position of point ‘Q’. What did you find ?

11) From your observations in question (10), complete this equation: tan(–x) = ______________________

12) How many values of ‘x’ satisfy the equation: tanx = tan(π/8) ? ______________________
(Hint: The function: y = tanx goes to infinity in both directions.)

13) Make observations with the unit circle, then complete these equations:

tan(π+x) = ______________________ tan(π-x) = ______________________

14) Sketch a graph of the function: y = tan(x) on the grid below. First mark the points where the function cuts the ‘x’ axis, then use the unit circle to help locate some intermediate points. Finally, join the points with smooth lines.