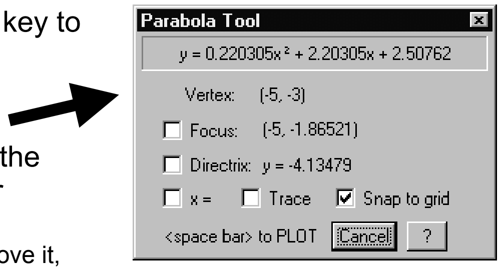
Quadratic Functions - Activity 1

An investigation of 'a' and 'c' in the equation: '$y = ax^2 + bx + c$'

1) Start Maths Helper Plus or hold down Ctrl and press the 'N' key to make a new document.



2) Select the 'Parabola' command from the 'Tools' menu. The parabola tool dialog box will appear on the screen. Make sure the 'Snap to grid' option is selected on the dialog box, but no other options.
If necessary, move the dialog box so that you can see all of the graph. To move it, point the mouse cursor to the words 'Parabol Tool' in its title bar and drag.

3) Move the mouse cursor over the graph view, and the curved parabola line will appear.
The quadratic equation at the top of the parabola tool dialog box is the equation of the parabola line.

4) To change the shape of the parabola, drag with the mouse in any direction.
Try dragging the mouse from side to side, then up and down.
HINT: For fine adjustment, hold down a 'Shift' key while you drag.

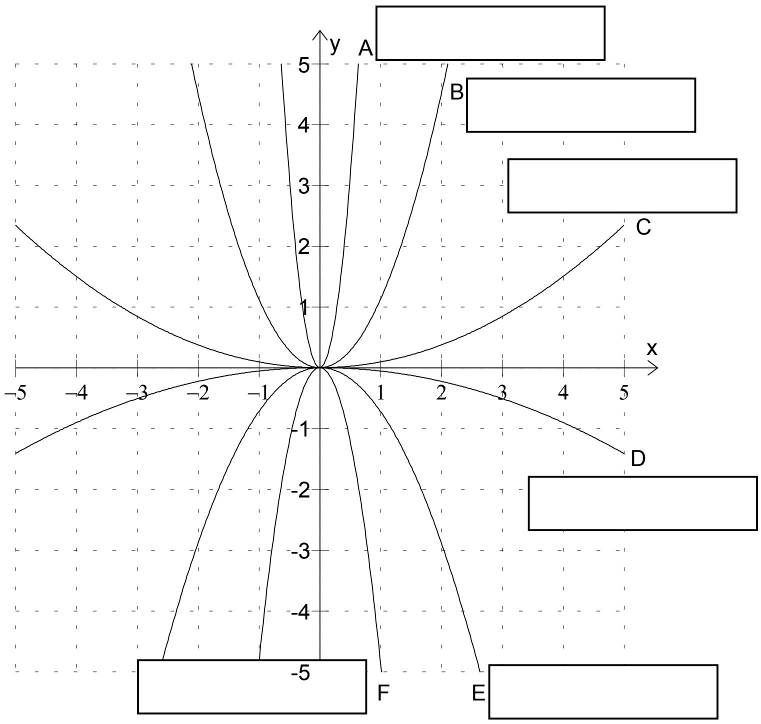
5) If 'b' and 'c' are zero, then the quadratic function is: $y = ax^2 + 0x + 0$, or simply: $y = ax^2$.

Move the parabola tool cursor so that it is centred on the origin. The quadratic equation will now look like this: '$y = ax^2$'.

By dragging the mouse, change the shape of the parabola so that it looks like curve 'A' on this diagram.

Write the 'a' value from the quadratic equation in the box near 'A' on the diagram.

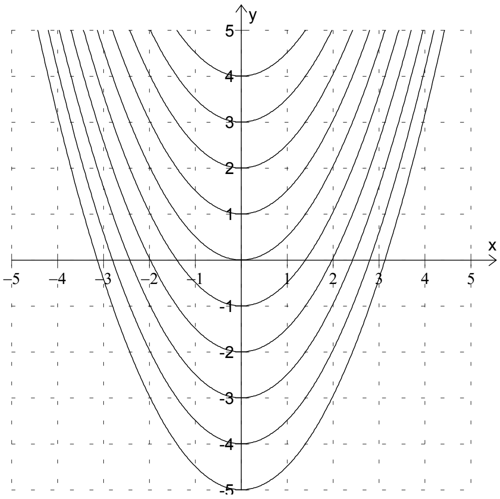
Repeat for each of the other graphs on this diagram, writing the 'a' values in the boxes each time.



6) How does the sign of the 'a' value effect the shape of a quadratic function graph ?

7) How does the size (called the magnitude) of the 'a' value effect the shape of a quadratic function graph ?

8) Move the parabola tool cursor over the graph view and centre it on the origin. Drag with the mouse until the quadratic equation displayed on the parabola tool dialog box is as close as possible to ‘$y = 0.5x^2$’.
Now move the parabola tool cursor to each of these positions, centred along the ‘y’ axis:



For each position, compare the value of ‘c’ in the parabola equation: ‘$y = ax^2 + c$’ with the ‘y’ intercept of the graph.

9) How does the ‘c’ value in the quadratic function: $y = ax^2 + c$ effect the position of its graph ?

10) Use what you have learned to make rough sketches of the graphs of the following quadratic functions:

(a) $y = x^2$

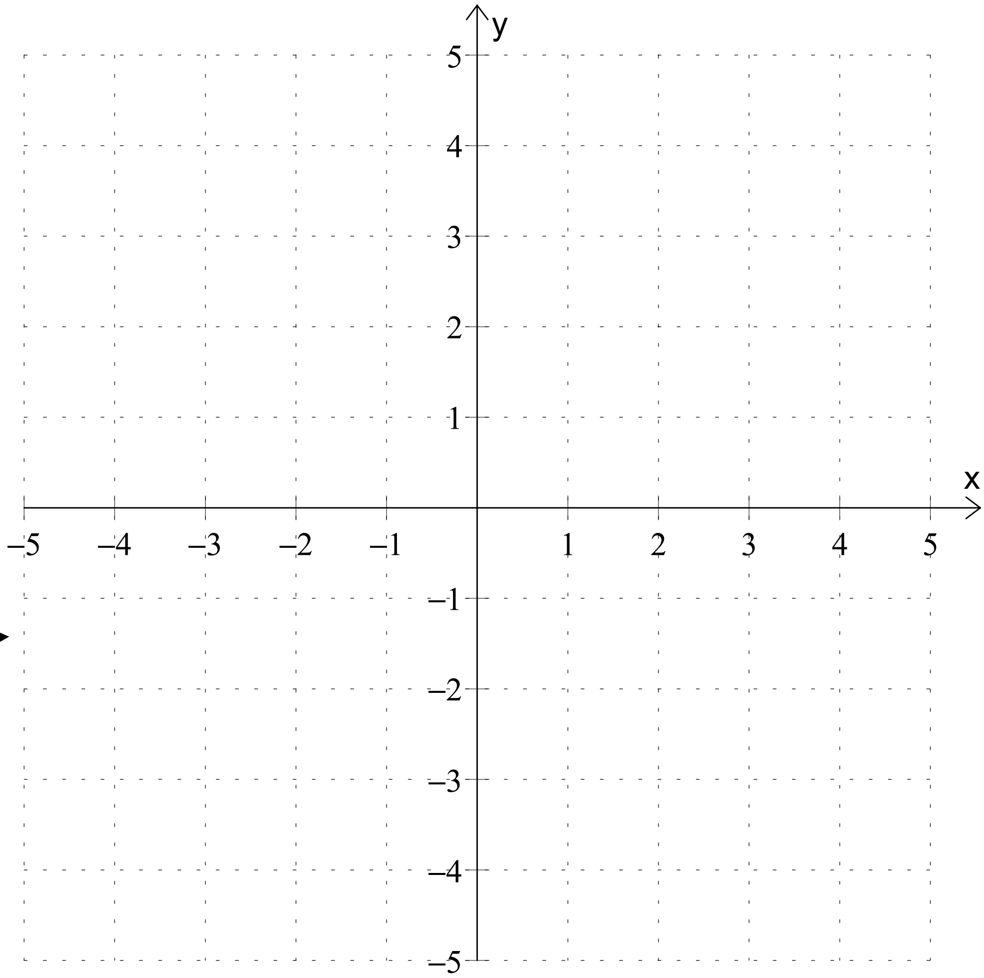
(b) $y = -2x^2$

(c) $y = 0.2x^2 - 4$

(d) $y = 20x^2 + 2$

(e) $y = -x^2 + 5$

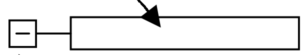
Use pencil to sketch your graphs on this graph grid:



11) Use Maths Helper Plus to test your answers. Graph each of these equations and compare the graphs with your sketches. Modify any of your sketches that are very different to the correct graphs.

To graph an equation you:

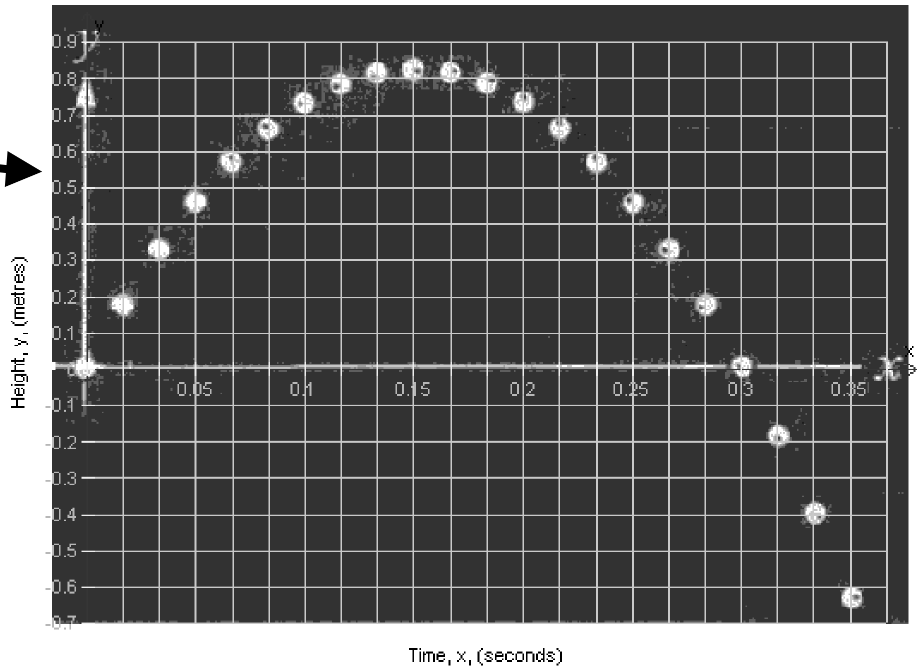
- Click on the input box. (On the text view.)
- Type the equation you want to check.
- Click outside of the input box.



Quadratic Functions - Activity 2

Applications of quadratic functions.

1) Start Maths Helper Plus and load the document:
'Parabolas - strobe picture.mhp'.
The graph view will display this strobe photograph of a moving ball:
The ball was projected into the air from the origin, and the camera flash recorded the ball's position at 1/60 th second intervals.



2) Select the 'Parabola' command from the 'Tools' menu. The parabola tool dialog box will appear on the screen. <u>Make sure the 'Snap to grid' option in the dialog box is NOT selected.</u>
If necessary, move the dialog box so that you can see all of the graph.
To move it, point the mouse cursor to the words 'Parabol Tool' in its title bar and drag.

3) Move the mouse cursor over the graph view, and the curved parabola line will appear. The quadratic equation at the top of the parabola tool dialog box is the equation of the parabola line. You can change the shape of the parabola graph by dragging with the mouse in any direction. Try dragging the mouse from side to side, then up and down.
HINT: For fine adjustment, hold down a 'Shift' key while you drag.

4) Can you fit a quadratic equation to the projectile path in the photograph ?__________

5) What is the approximate equation for the path of this projectile ?_________________________

6) You can use the parabola tool to plot the quadratic equation for the projectile. Position the parabola to go as close as possible through the centre of each ball image, then press the space bar to plot the equation.

7) What is the maximum height reached by the projectile ?___________________________

8) When is the projectile at a height of 0.9 m ? _____________________________________

9) When is the projectile at maximum height ?______________________________________

10) When is the projectile at a height of 0.45 m ?_____________________________________

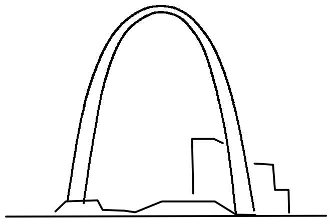
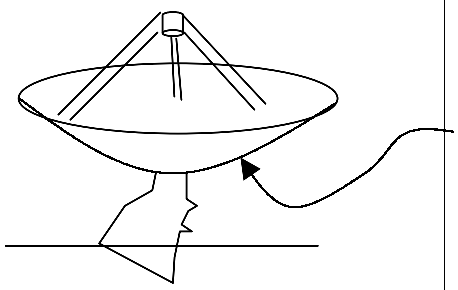
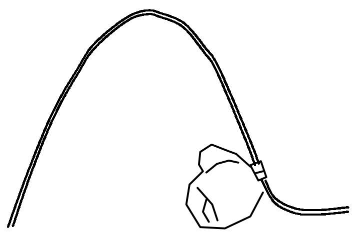
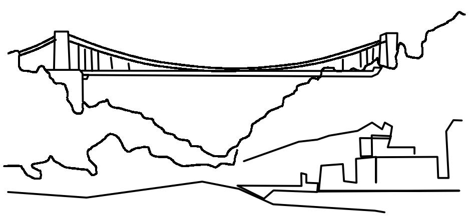
Projectile motion near the Earth is only one of many applications of quadratic equations.

In general, quadratic equations can have up to two solutions. Sometimes they have only one solution and sometimes they have none. Questions 8,9 and 10 demonstrate this.

The shape of a quadratic function graph is called a ‘parabola’. Parabolas often appear in everyday life both naturally and in man-made structures. Some things that look like parabolas may not be parabolas at all. A good way to test a shape to see if it is a parabola is to try fitting a quadratic function to it. With Maths Helper Plus this is easy.

Load the Maths Helper Plus document: ‘Curve pictures.mhp’

Select the parabola tool, then try to fit a quadratic function curve to curved shapes found in each picture. Plot the parabolas by pressing the space bar. Use your observations to answer the questions below:

Object	
 The Gateway Arch, St Louis.	Is the Arch a parabola?
 Parkes radio telescope, Australia.	Can you fit a parabola to the radio telescope antenna outline? (Hint, the actual antenna surface is made of finer mesh. The more open structures making up the bottom part of the dish are not part of the antenna.) Write the equation of the parabola below:
 Water from garden hose.	Fit a parabola to the water stream. Write its equation below: Why would you expect the water stream to form a parabola ? (Hint, look at the picture on the front of this sheet.)
 Clifton Suspension Bridge, Bristol	Can you fit a parabola to the cables supporting the suspension bridge? Even though it seems to fit well, this curve is not really a parabola at all. It is a special function called a ‘catenary’. (The Gateway Arch is also a catenary.) Find out about catenary functions and where else they appear in our surroundings.

Quadratic Functions - Activity 3

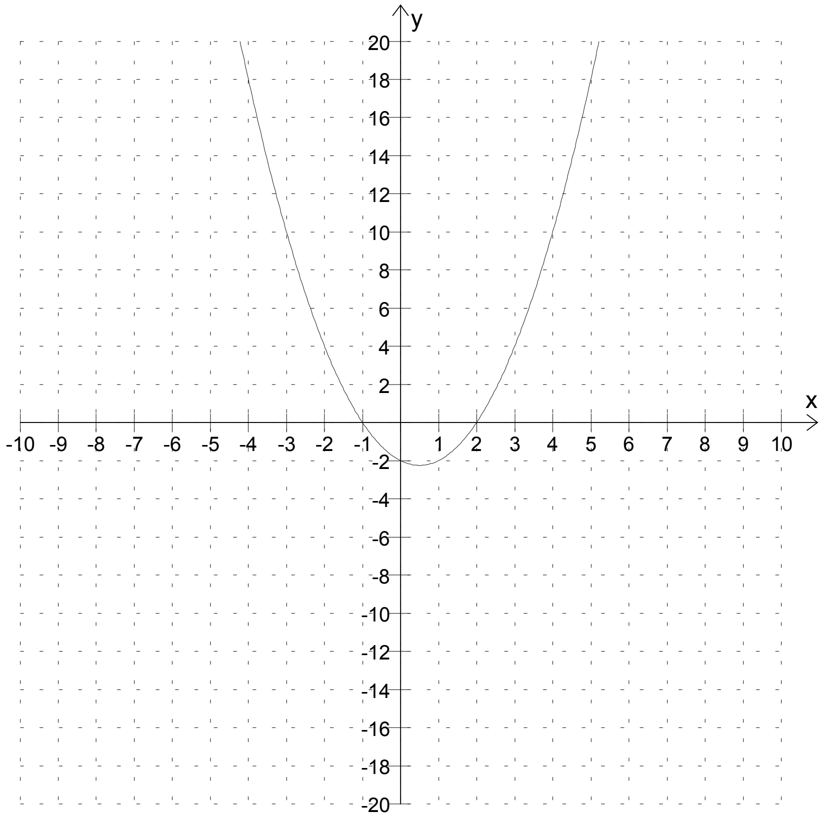
Evaluating, finding points that satisfy, and sketching quadratic functions.

1) Start Maths Helper Plus and load the document: 'Quadratic - table of values.mhp'.

This document displays a data table and graph for the quadratic function: $y = Ax^2+Bx+C$
(In this case, A = 1, B = –1 and C = –2, so the actual quadratic function is: $y = x^2 - x - 2$)

y = Ax²+Bx+C

x	Ax²	Bx	C	y
-10	100	10	-2	108
-9	81	9	-2	88
-8	64	8	-2	70
-7	49	7	-2	54
-6	36	6	-2	40
-5	25	5	-2	28
-4	16	4	-2	18
-3	9	3	-2	10
-2	4	2	-2	4
-1	1	1	-2	0
0	0	0	-2	-2
1	1	-1	-2	-2
2	4	-2	-2	0
3	9	-3	-2	4
4	16	-4	-2	10
5	25	-5	-2	18
6	36	-6	-2	28
7	49	-7	-2	40
8	64	-8	-2	54
9	81	-9	-2	70
10	100	-10	-2	88



2) To 'evaluate' a quadratic function means to calculate the 'y' value of the function from a given 'x' value. A quadratic expression: Ax^2+Bx+C has three terms: Ax^2, Bx and C that are added to give 'y'. In the example above, A = 1, B = –1 and C = –2, so for this quadratic function:

$$\begin{aligned} y &= 1x^2 + -1x + -2 \\ &= x^2 - x - 2 \end{aligned}$$

For example, evaluate this quadratic function at 'x' = 3. if 'x' = 3, then 'y' = $3^2 - 3 - 2 = 9 - 3 - 2 = 4$
Look at the row of the table (above) where 'x' = 3. Notice the three terms Ax^2, Bx and C evaluated separately, and their sum, 4, listed as the 'y' value.

Without looking at the table created by Maths Helper Plus, evaluate the quadratic function for the 'x' values in this table. First calculate the separate terms, then add the terms together to find the 'y' value:

x	Ax² = 1x²	Bx = –1x	C = –2	y = Ax²+Bx+C
10				
5				
2				
-1				
-4				
-9				

Now correct your work by looking at the data table produced by Maths Helper Plus.

Why can a quadratic function have different ‘x’ values that give the same ‘y’ value ?

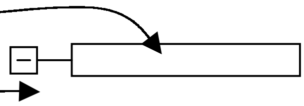
3) When a function is evaluated at a given ‘x’ value and a ‘y’ value is obtained, then the point (x,y) is said to ‘satisfy’ it. For the ‘x’ values in the table above, write (x,y) points that satisfy the quadratic function: $y = x^2 - x - 2$

____

4) Plot these points on the graph in Maths Helper Plus.

To plot points you:

- Click on the input box. (On the text view.)
- Type the points to plot, like this: (1,2) (2,3) (3,4)
- Click outside of the input box.



5) Look at the plotted points on the graph view. What does their position have to do with the graph of the quadratic function ?

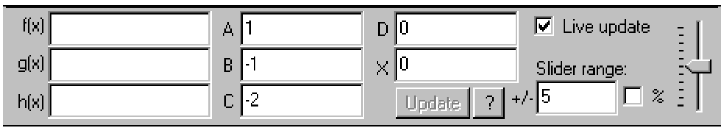
6) Complete this table for the quadratic function where A = -1, B = 2 and C = 15:

x	Ax^2 =	Bx =	C =	$y = Ax^2+Bx+C$
7				
5				
3				
1				
-1				
-3				
-5				

For these ‘x’ values, list the (x,y) points that satisfy the quadratic function: $y = -x^2 + 2x + 15$

7) Plot these points on the graph on the front of this sheet. If they satisfy the equation, then they will lie on the equation graph. Sketch the parabola curve through the points. Use pencil and draw a smooth curved line.

8) Press the F5 key to display the parameters box if not already displayed:



You use the parameters box to set values of A, B and C to make different quadratic functions.
To change the value of A, B or C, you click on the edit box near the letter name, make your changes, then click the ‘Update’ button. The graph and table will change immediately.

9) Use the parameters box to set A = -1, B = 2, C = 15. Correct your own table and graph sketch using the table and graph displayed by Maths Helper Plus.

Quadratic Functions - Activity 4

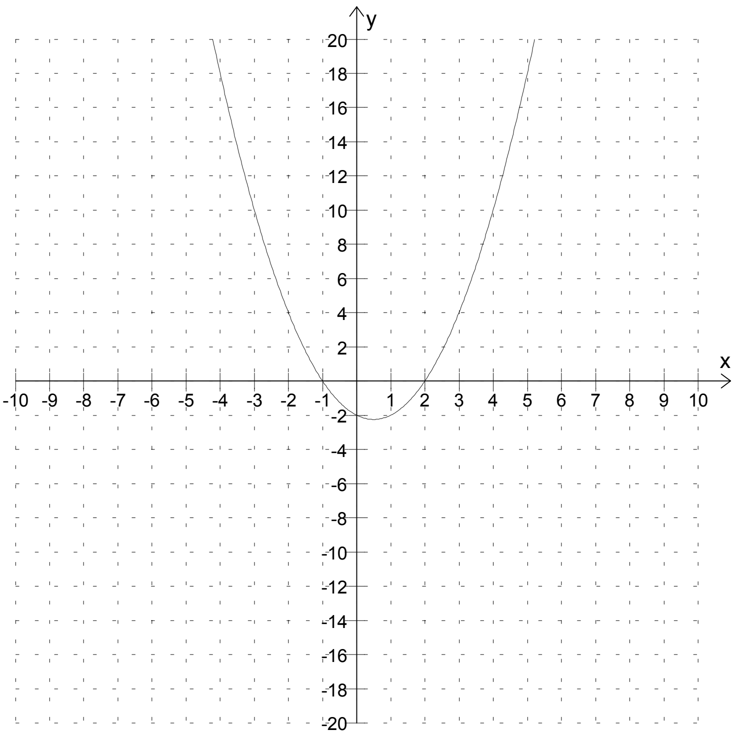
Finding zeros and roots.

1) Start Maths Helper Plus and load the document: 'Quadratic - table of values.mhp'.

This document displays a data table and graph for the quadratic function: $y = Ax^2+Bx+C$
(In this case, A = 1, B = –1 and C = –2, so the actual quadratic function is: $y = x^2 – x – 2$)

$y = Ax^2+Bx+C$

x	Ax²	Bx	C	y
-10	100	10	-2	108
-9	81	9	-2	88
-8	64	8	-2	70
-7	49	7	-2	54
-6	36	6	-2	40
-5	25	5	-2	28
-4	16	4	-2	18
-3	9	3	-2	10
-2	4	2	-2	4
-1	1	1	-2	0
0	0	0	-2	-2
1	1	-1	-2	-2
2	4	-2	-2	0
3	9	-3	-2	4
4	16	-4	-2	10
5	25	-5	-2	18
6	36	-6	-2	28
7	49	-7	-2	40
8	64	-8	-2	54
9	81	-9	-2	70
10	100	-10	-2	88



2) The 'zeros' of a quadratic function $y = ax^2 + bx + c$ are the 'x' values for which 'y' is zero. From the table above, what are the zeros of this quadratic function?

x =__________ and x =__________

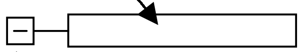
3) Write the 'x' and 'y' values of each zero as an ordered pair:

zero 1: (_____ , _____) zero 2: (_____ , _____)

4) Plot these two points on the graph above, then correct your work by plotting them in Maths Helper Plus.

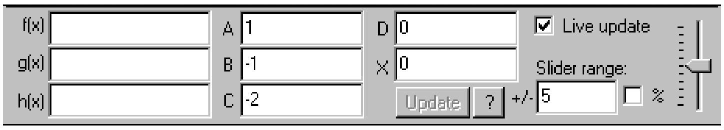
To plot points you:

- Click on the input box. (On the text view.)
- Type the points to plot, like this: (1,2) (2,3)
- Click outside of the input box.



5) How could you find the zeros of a quadratic function from a graph of the function ?

6) Press the F5 key to display the parameters box if not already displayed:



You use the parameters box to set values of A, B and C to make different quadratic functions. To change the value of A, B or C, you click on the edit box near the letter name, make your changes, then click the 'Update' button. The graph and table will change immediately.

For each equation in the table below:

- Hide the text view by holding down a 'Ctrl' key and pressing the 'G' key,
- Change the A, B and C values in the parameters box to plot the quadratic function,
- Obtain the zeros of the quadratic function from the graph and then write them in the table.
- Display the text view by holding down a 'Ctrl' key and pressing the 'T' key,
- Use the values in the table on the text view to correct your answers.

Quadratic Function	Zero 1, x =	Zero 2, x =
$y = x^2 + x - 12$		
$y = x^2 - 3x - 54$		
$y = x^2 + 10x + 25$		
$y = x^2 + x - 56$		
$y = x^2 + 7x$		

7) If a quadratic function has only one zero, describe how you could tell this simply by inspecting its graph ?

8) If a quadratic function has no zeros, describe how you could tell this simply by inspecting its graph ?

The quadratic equation: $Ax^2+Bx+C = 0$ is satisfied by the zeros of the corresponding quadratic function. These zeros located on the function graphs are called <u>real roots</u> of this quadratic equation. Thus there may be 2, 1 or 0 real roots of a quadratic equation. (Quadratic equations may also have imaginary roots that cannot be found from intersections on the graphs.)

9) Use Maths Helper Plus to predict how many real roots these quadratic equations have:
[HINT: Plot the corresponding quadratic <u>functions</u>.]

Quadratic equation:	Number of real roots:
$x^2 + 8x + 16 = 0$	
$x^2 - 12x + 42 = 0$	
$-5x^2 - 70x - 245 = 0$	
$2x^2 - 16x + 24 = 0$	
$x^2 = 0$	

Name:______________________________________ Class: ___________

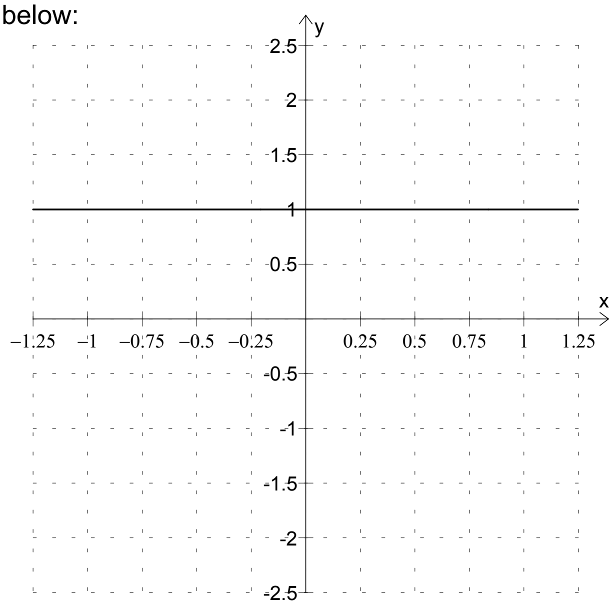
Polynomials - Activity 1

Investigating graphs of functions of type: $y = x^n$.

1) Start Maths Helper Plus, then load the file: 'Polynomials 1.mhp'.

The graph view displays a plot of the function: $y = x^n$, where 'n' can be any whole number value. In Maths Helper Plus constants cannot be named 'n', so 'A' will be used instead. Thus we have: $y = x^A$

Initially, 'A' = 0, so the plotted function is: $y = x^0$ as shown below:



2) Explain the shape of this graph.

3) Predict the shape of the graph of $y = x^A$ when 'A' = 1.
Sketch the curve you predict on this graph:

4) Repeat for 'A' = 2.
Predict and sketch the graph, as for '3' above.

5) Repeat for 'A' = 3.
Predict and sketch the graph, as for '3' above.

6) You can use Maths Helper Plus to correct your answers to questions 3, 4 and 5 above.

Press F5 to display the parameters box, then...

a) Click here to select variable 'A'

b) Click here to select the slider

With the slider selected, you can use the up and down arrow keys on the keyboard to change the 'A' values. The up arrow key will increase 'A' by 1, while the down arrow key will decrease 'A' by 1.
If the arrow keys stop responding, click the slider button again.

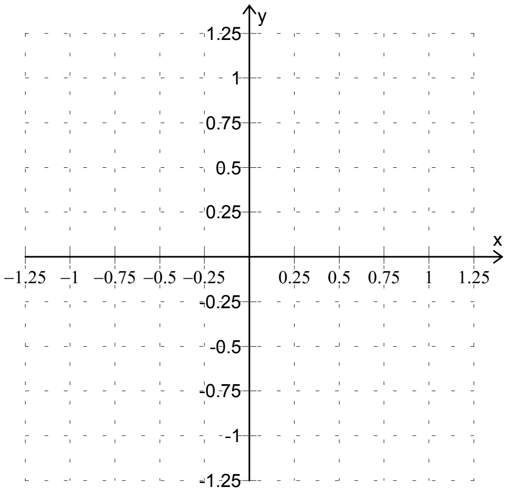
Set 'A' to 1, then 2 and then 3. For each value of 'A', compare the function curve in Maths Helper Plus with your answers above. Correct your work if necessary.

7) For each of the four graph grids below, set 'A' to the given value, complete the function equation, and copy the plotted function curve from Maths Helper Plus:

a) A = 4

Equation:

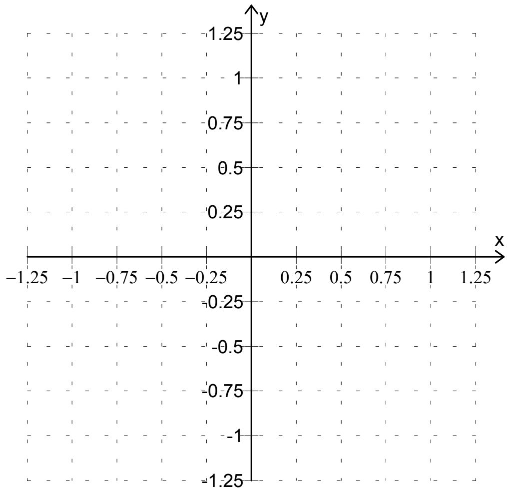
y = x☐



b) A = 5

Equation:

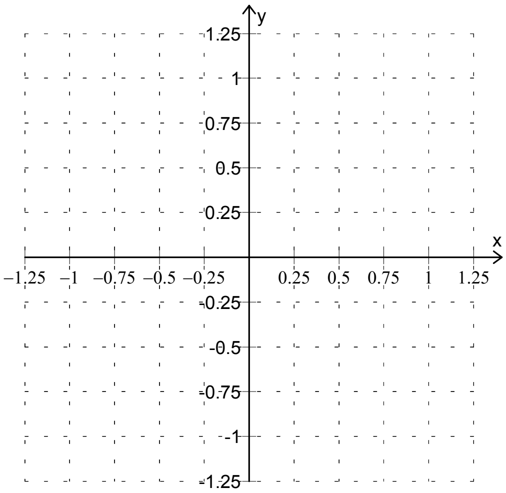
y = x☐



c) A = 6

Equation:

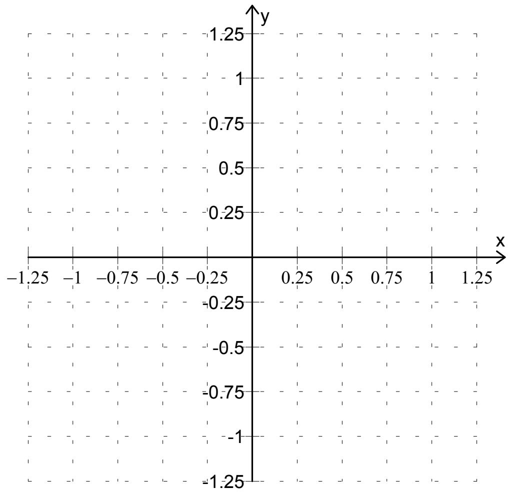
y = x ☐



d) A = 7

Equation:

y = x ☐



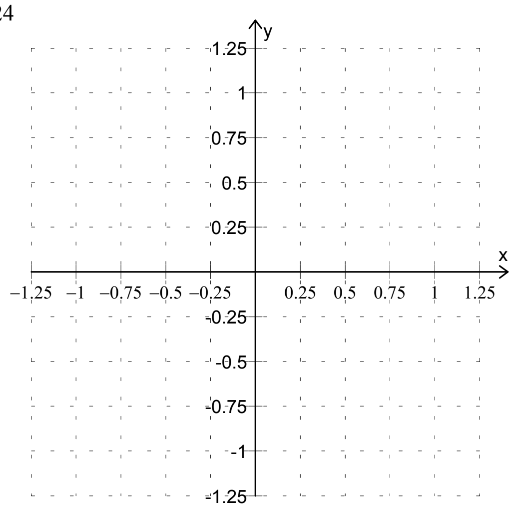
8) What pattern(s) do you observe in the graph curves of $y = x^n$ functions that you have plotted ?

9) For each of the following equations,

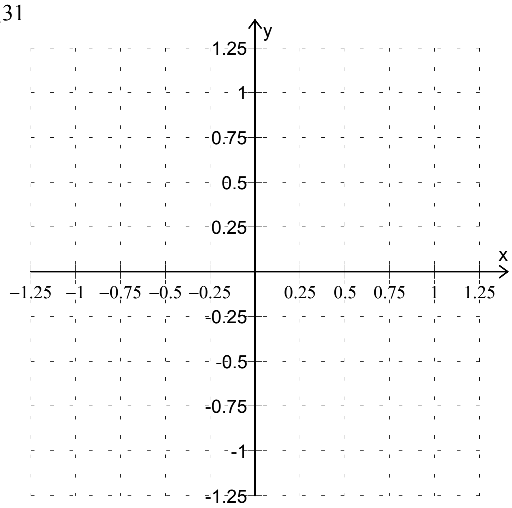
- sketch the function curve for the equation
- use Maths Helper Plus to correct your work.

[HINT: To set 'A' to a given value without using the arrow keys, click on the 'A' edit box, type the value of 'A', then click the 'Update' button on the parameters box.]

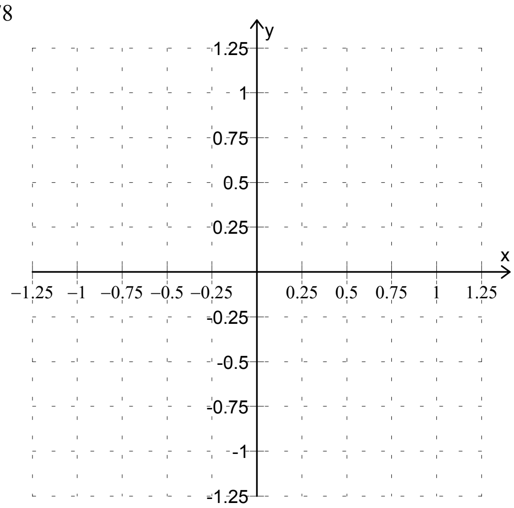
a) $y = x^{24}$



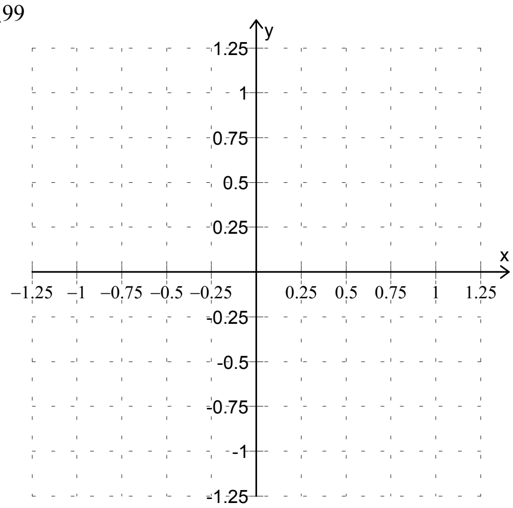
b) $y = x^{31}$



c) $y = x^{78}$



d) $y = x^{99}$



Polynomials - Activity 2

Degree, turning points and zeros of polynomials.

General information about polynomials:

A polynomial is an expression having terms with decreasing powers of 'x', like this:

$$2x^3 + 3x^2 - x + 6$$

If: ax^n is a term of a polynomial, then 'a' is the coefficient of the term, and 'n' is the index of the term. Polynomials are usually written so that there is only one term with a given index.

Technically, there must be more than one term in a polynomial (poly = many), but missing terms can be included by using zero coefficients, like this: $x^3 \Rightarrow x^3 + 0x^2 + 0x + 0$

1) Name the coefficient and the index for each of the four terms in: $2x^3 + 3x^2 - x + 6$

Term	Coefficient	Index
$2x^3$		
$3x^2$		
$-x$		
6		

The maximum index in a polynomial is called the degree of the polynomial, thus $5x^2 - 3x + 1$ is a second degree polynomial, and $3x^4 + 7x^3 - 2x^2 + 9x - 20$ is a fourth degree polynomial.

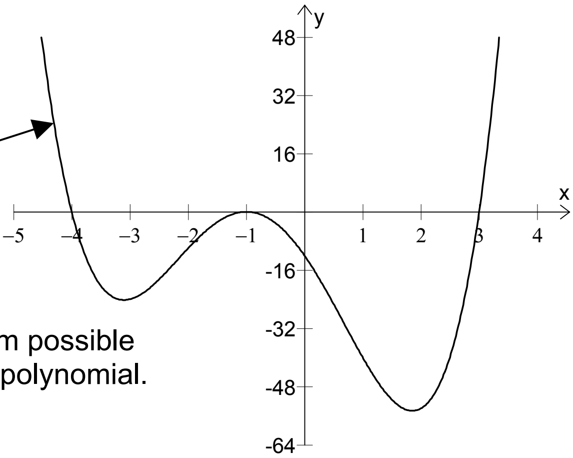
Second degree polynomials are also called quadratic polynomials, and third degree polynomials are also called cubic polynomials.

Polynomial functions and their graphs:

This is an example of a polynomial function:

$$y = x^4 + 3x^3 - 9x^2 - 23x - 12$$

The graph of this polynomial function is as follows:



In general, polynomial function graphs consist of a smooth line with a series of hills and valleys.

The hills and valleys are called turning points. The maximum possible number of turning points is one less than the degree of the polynomial.

2) What is the degree of this polynomial ?

3) Does it have the maximum possible number of turning points for polynomials of this degree ?

Zeros of polynomial functions:

A zero of a polynomial function is an 'x' value for which 'y' = 0. At these 'x' values, its graph cuts or touches the 'x' axis. The maximum number of zeros of a polynomial is the same as its degree.

4) For the polynomial function $y = x^4 + 3x^3 - 9x^2 - 23x - 12$ graphed above,

a) What are its zeros ?

b) Does it have the maximum number of zeros for a polynomial function of this degree ?

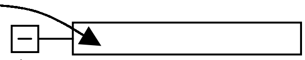
You cannot be exactly sure of values you read from a printed graph. Maths Helper Plus can find zeros and turning points of polynomial functions with very high accuracy.

Perform the steps (a) to (c) below to practice finding accurate zeros and turning points of polynomials.

For practice, use the polynomial function: $y = x^4 + 3x^3 - 9x^2 - 23x - 12$.

(a) Graph the polynomial function in Maths Helper Plus. Use 'Ctrl'+N to make a new document, then:

- Click on the input box. (On the text view.)
- Type the polynomial function.
- Click outside of the input box.



To type the index of a polynomial term, use '^', eg: `3x^3` means $3x^3$.

So to enter: $y = x^4 + 3x^3 - 9x^2 - 23x - 12$ you type: `y = x^4 + 3x^3 - 9x^2 - 23x - 12`

[**HINT**: For tidier looking equations, use the '³' and '²' symbols available on the short cuts box, instead of ^2 and ^3.]

- Adjust the vertical scale if all turning points are not visible, like this: Hold down a 'Ctrl' key and click the down arrow key enough times to see all of the turning points. 'Ctrl' + up arrow reverses this operation.

(b) Locate zeros at intersection points between the graph and the 'x' axis with the intersection tool. Click the toolbar button, then click the mouse cursor on the graph where the graph intersects the 'x' axis. Read the 'x' coordinate from the dialog box, this is a zero of the function.

- Use the intersection tool to locate zeros of $y = x^4 + 3x^3 - 9x^2 - 23x - 12$.

(c) Locate zeros at turning points on the graph, with the turning point tool. Click the toolbar button, then click the mouse cursor on the graph at a turning point. If a turning point is found, the dialog box will display its 'x' and 'y' coordinate. This 'x' coordinate is a 'zero' of the function only if the 'y' value of the turning point = 0.

- Use the turning point tool to locate the zero of $y = x^4 + 3x^3 - 9x^2 - 23x - 12$ that lies on a turning point.

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5) For each of the polynomial functions in the table below...

a) Determine the maximum possible number of zeros and turning points. Write your answers in the table. (' max zeros' and 'max t.p.')

b) Graph the function in a new Maths Helper Plus document.

c) Record the actual number of tuning points in the table. ('actual t.p.')

d) Locate and record all zeros of the function with the intersection and turning point tools.

Polynomial function	max zeros	max t.p.	actual t.p.	zeros at x =
$y = x^2 - 3x - 4$				
$y = 0.03x^3 - 0.06x^2 + 0.6x + 1.6$				
$y = 0.03x^4 + 0.03x^3 - 0.3x^2 - 0.2x + 1$				
$y = 0.02x^5 - 0.04x^4$ $- 0.03x^3 + 0.04x^2 - 2x + 1$				
$y = x^5 + 3x^4 - 22x^3 - 56x^2 + 96x + 128$				