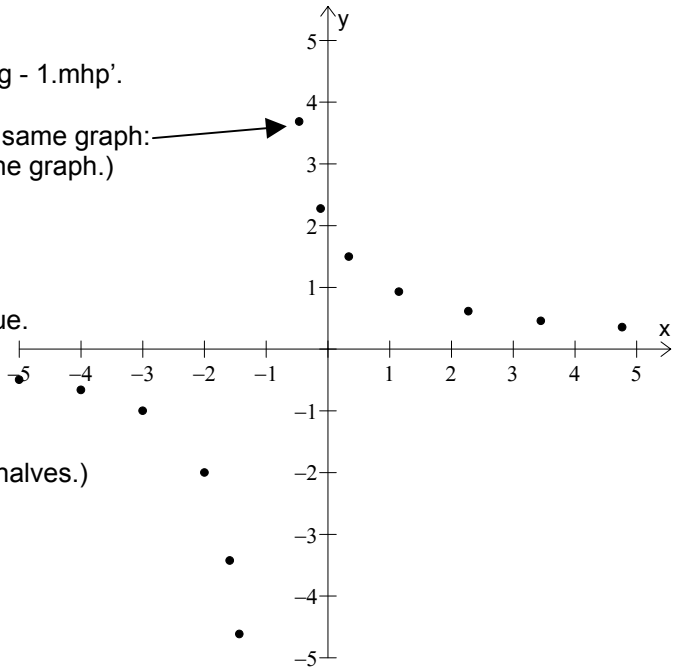
Curve Fitting - Activity 1

Finding the equation of type: $y = \frac{a}{bx + c}$ that fits given data points by fitting a straight line.



1) Start Maths Helper Plus, then load the file: 'Curve fitting - 1.mhp'.

The graph view displays five sets of plotted points on the same graph:
(This diagram only shows point set 'a', plotted in red on the graph.)

The points lie on the line of an unknown function of 'x'.
It is our task to find this unknown function.

The shape of the outline of the points gives us the first clue.
It has two halves that go in opposite directions and are not connected.

2) Use a pencil to join the dots for each half of the set of points on this diagram with a smooth line. (Don't join the halves.)

This kind of shape:  or this: 

immediately indicates a function related to $y = \frac{1}{x}$.

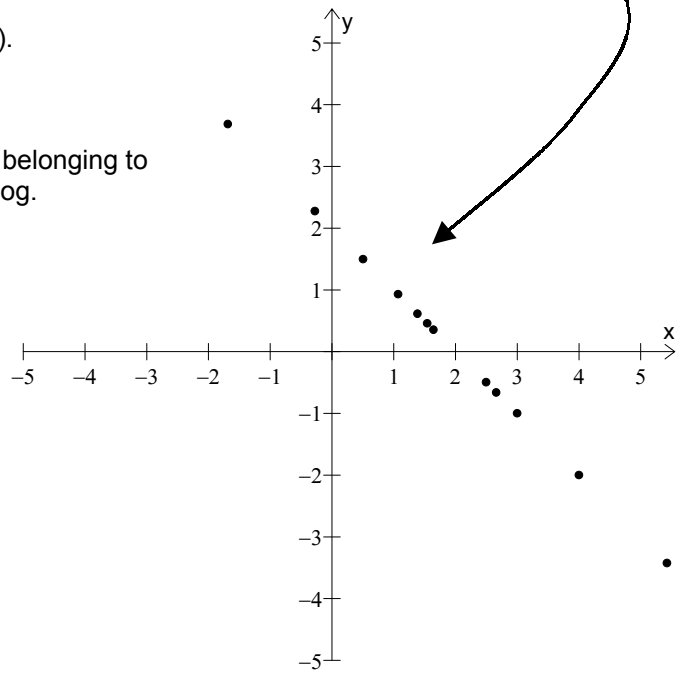
Here are some examples of these kinds of functions: $y = \frac{15}{x}$, $y = \frac{-7}{11x}$, $y = \frac{1}{9 - 6x}$, $y = \frac{5}{7x + 10}$

To find the function that the plotted points lie on, the 'x' or 'y' coordinates of the plotted points can be modified according to some known rule such that the points then plot as a straight line. (See graph below:) From the equation of the straight line and the rule, the function that fits the plotted points can be found.

For functions related to $y = \frac{1}{x}$, find the function that fits the points like this:

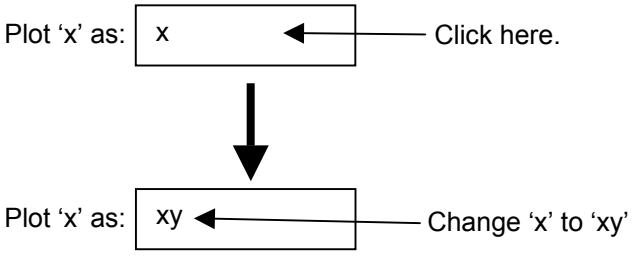
- Replace 'x' with 'xy'.
- Find the equation of the straight line through the points, but write it down with a capital 'X'.
- Write the equation again, but replace 'X' with 'xy'.
- Simplify the equation so that it is in the form: y = f(x).

Let's find the function that fits the points in the graph above.



3) With the mouse pointer near the centre of a plotted point belonging to point set 'a', carefully double click to display the options dialog.

The '(x,y) Point Settings' tab of the dialog will be displayed.
Near the bottom of the dialog box, click on the edit box for 'Plot 'x' as:' and change the 'x' to 'xy':



Click 'OK' to close the dialog box.
The points will now be in a straight line.

4) Rule a line through the exact centre of these points

- For point set ‘a’, rule a line on the diagram at the bottom of the front page of this sheet.

> For the other sets of points, If you can’t make a printed copy then make a line through the dots on your computer screen, like this:
> - With the mouse pointer near the centre of a plotted point, carefully double click as before to display the options dialog.
> - Click to select the ‘display’ option for ‘Joining Lines’.
> - Click OK to close the dialog box.

Find the equation of the straight line. Use ‘X’ instead of ‘x’ in your equation because it really means ‘xy’.

- For point set ‘a’, show that the equation of the line is: y = -X + 2

5) Replace ‘X’ in the equation with ‘xy’.

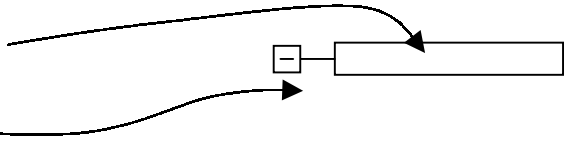
- For point set ‘a’ we have: y = -xy + 2. Show that this simplifies to: $y = \frac{2}{x+1}$

6) To see if this is correct, plot this equation in Maths Helper Plus. If the points are accurate, the function line will pass through each one.

(First you will need to reverse the changes you made in steps 3 and 4 above by double clicking on a plotted point, changing ‘xy’ back to ‘x’, then turning off the joining lines.)

To plot the equation you:

- Click on the input box. (On the text view.)
- Type the equation you want to check.
- Click outside of the input box.



You must put the denominator in brackets if it has more than one term or factor.

- For point set ‘a’, the denominator has two terms, so you enter $y = \frac{2}{x+1}$ like this: y = 2 / (x+1)

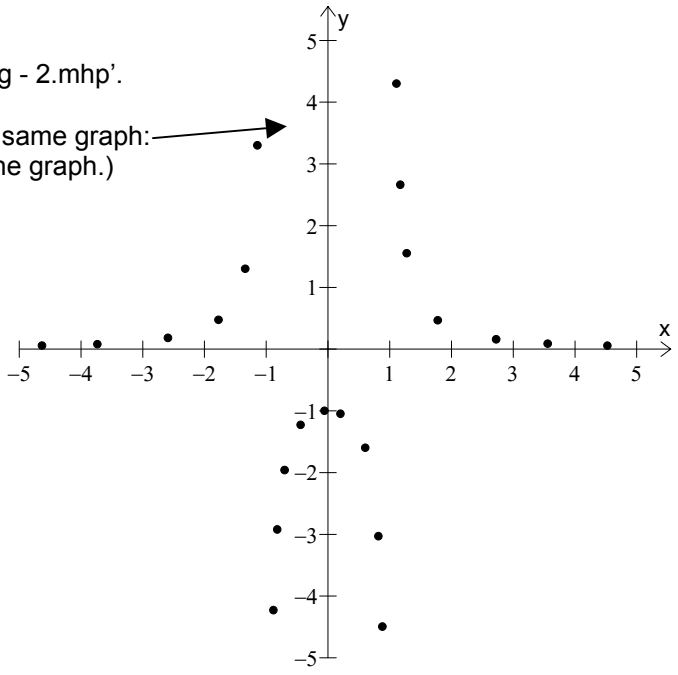
If your equation is correct, then the function curve will pass through all points in a given set.

Use the steps described above to find functions that fit each of the other sets of plotted points.
Record your answers in the table below: (Point set ‘a’ has already been completed.)
Graph each function to make sure it passes through each of the original points.

Set of points:	Straight line equation in ‘X’	Replace ‘X’ with ‘xy’	Simplify
‘a’ (red dots)	y = -X + 2	y = -xy + 2	$y = \frac{2}{x+1}$
b (blue **+**)			
c (green circles)			
d (pink dots)			
e (black circles)			

Curve Fitting - Activity 2

Finding the equation of type: $y = \frac{a}{bx^2 + c}$ that fits given data points by fitting a straight line.



1) Start Maths Helper Plus, then load the file: 'Curve fitting - 2.mhp'.

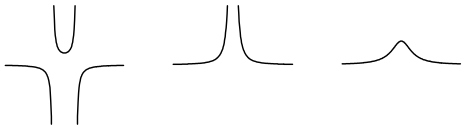
The graph view displays five sets of plotted points on the same graph:
(This diagram only shows point set 'a', plotted in red on the graph.)

The points lie on the line of an unknown function of 'x'.
It is our task to find this unknown function.

This graph has three separate parts that are symmetrical about the 'y' axis.

2) Use a pencil to join the dots for each part of this set of points on this diagram with a smooth line.
(Don't join the three parts to each other.)

These kinds of shapes:



immediately suggest a function related to $y = \frac{1}{x^2}$.

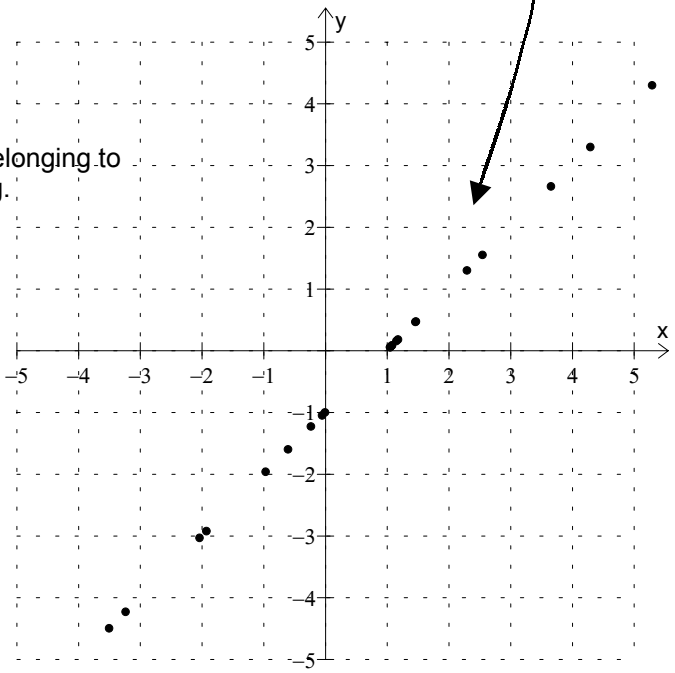
Here are some examples of these kinds of functions: $y = \frac{3}{x^2}$, $y = \frac{2}{x^2 + 3}$, $y = \frac{1}{9 - 6x^2}$, $y = \frac{-5}{2x^2 + 1}$

To find the function that the plotted points lie on, the 'x' or 'y' coordinates of the plotted points can be modified according to some known rule such that the points then plot as a straight line. (See graph below:) From the equation of the straight line and the rule, the function that fits the plotted points can be found.

For functions related to $y = \frac{1}{x^2}$, find the function that fits the points like this:

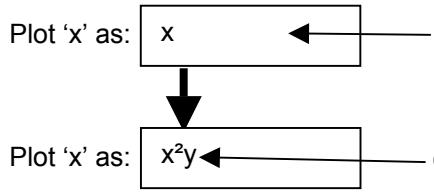
- Replace 'x' with 'x²y'.
- Find the equation of the straight line through the points, but write it down with a capital 'X'.
- Write the equation again, but replace 'X' with 'x²y'.
- Simplify the equation so that it is in the form: y = f(x).

Let's find the function that fits the points in the graph above.



3) With the mouse pointer near the centre of a plotted point belonging to point set 'a', carefully double click to display the options dialog.

The '(x,y) Point Settings' tab of the dialog will be displayed.
Near the bottom of the dialog box, click on the edit box for 'Plot 'x' as:' and change the 'x' to 'xy':



[HINT: Get the '²' from the shortcuts box, or use '^2' instead of '²', ie: 'x^2y'.]
Click 'OK' to close the dialog box.
The points will now be in a straight line.

4) Rule a line through the exact centre of these points

- For point set ‘a’, rule a line on the diagram at the bottom of the front page of this sheet.

> For the other sets of points, If you can’t make a printed copy then make a line through the dots on your computer screen, like this:
> - With the mouse pointer near the centre of a plotted point, carefully double click as before to display the options dialog.
> - Click to select the ‘display’ option for ‘Joining Lines’.
> - Click OK to close the dialog box.

Find the equation of the straight line. Use ‘X’ instead of ‘x’ in your equation because it really means ‘x²y’.

- For point set ‘a’, show that the equation of the line is: y = X - 1

5) Replace ‘X’ in the equation with ‘x²y’.

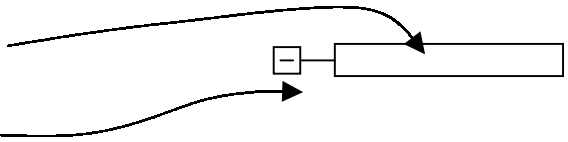
- For point set ‘a’ we have: y = x²y – 1. Show that this simplifies to: $y = \frac{1}{x^2 - 1}$

6) To see if this is correct, plot this equation in Maths Helper Plus. If the points are accurate, the function line will pass through each one.

(First you will need to reverse the changes you made in steps 3 and 4 above by double clicking on a plotted point, changing ‘x²y’ back to ‘x’, then turning off the joining lines.)

To plot the equation you:

- Click on the input box. (On the text view.)
- Type the equation you want to check.
- Click outside of the input box.



You must put the denominator in brackets if it has more than one term or factor.

- For point set ‘a’, the denominator has two terms, so you enter $y = \frac{1}{x^2 - 1}$ like this: y = 1 / (x²–1)

If your equation is correct, then the function curve will pass through all points in a given set.

Use the steps described above to find functions that fit each of the other sets of plotted points.
Record your answers in the table below: (Point set ‘a’ has already been completed.)
Graph each function to make sure it passes through each of the original points.

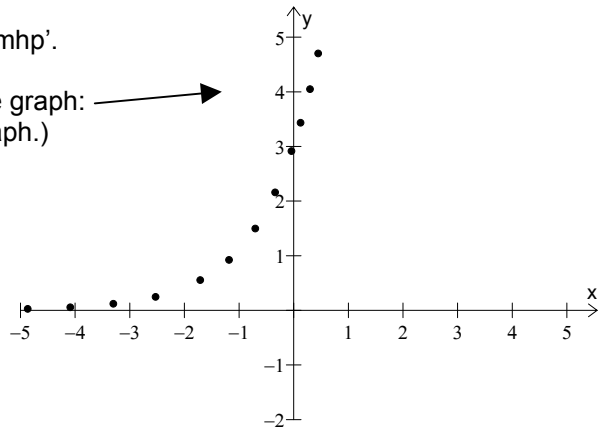
Set of points:	Straight line equation in ‘X’	Replace ‘X’ with ‘xy’	Simplify
‘a’ (red dots)	y = X – 1	y = x²y – 1	$y = \frac{1}{x^2 - 1}$
b (blue **+**)			
c (green circles)			
d (pink dots)			
e (black circles)			

Curve Fitting - Activity 3

Finding the equation of type: $y = ae^{bx}$ or $y = e^{ax+b}$ that fits given data points by fitting a straight line.

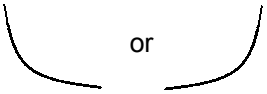
1) Start Maths Helper Plus, then load the file: 'Curve fitting - 3.mhp'.

The graph view displays five sets of plotted points on the same graph:
(This diagram only shows point set 'a', plotted in red on the graph.)



The points lie on the line of an unknown function of 'x'.
It is our task to find this unknown function.

2) Use a pencil to join the dots on this diagram with a smooth line.

These kinds of shapes:  or

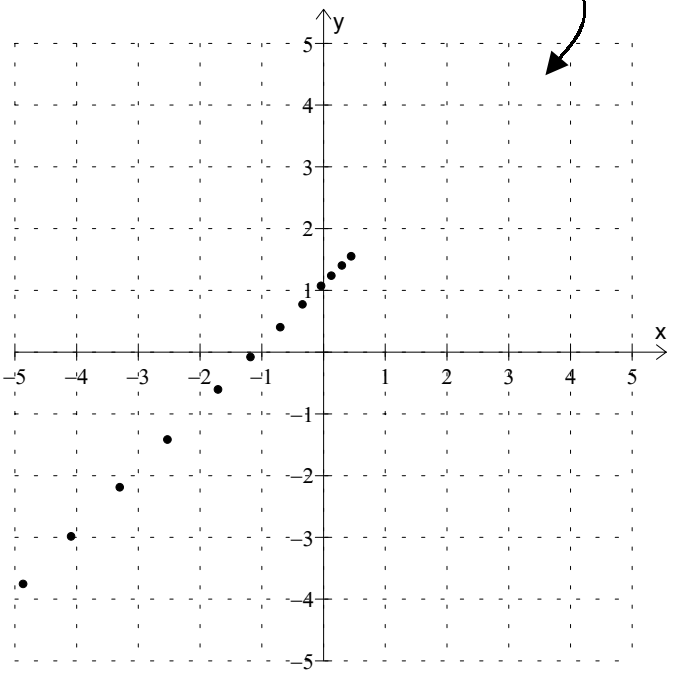
immediately suggest a function related to $y = e^x$.

Here are some examples of these kinds of functions: $y = 2e^x$, $y = 5e^{3x}$, $y = e^{-2x+1}$, $y = e^{4x-2}$

To find the function that the plotted points lie on, the 'x' or 'y' coordinates of the plotted points can be modified according to some known rule such that the points then plot as a straight line. (See graph below:) From the equation of the straight line and the rule, the function that fits the plotted points can be found.

For functions related to $y = e^x$, find the function that fits the points like this:

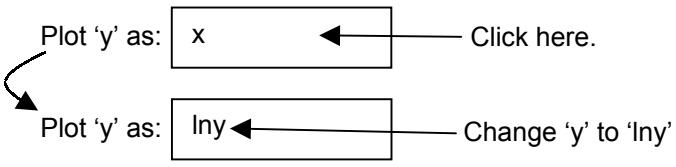
- Replace 'y' with 'lny'. ['lny' means 'natural log of y', or $\log_e y$]
- Find the equation of the straight line through the modified points, but write it down with a capital 'Y'.
- Write the equation again, but replace 'Y' with 'lny'.
- Simplify the equation so that it is in the form: y = f(x).



Let's find the function that fits the points in the graph above.

3) With the mouse pointer near the centre of a plotted point belonging to point set 'a', carefully double click to display the options dialog.

The '(x,y) Point Settings' tab of the dialog will be displayed.
Near the bottom of the dialog box, click on the edit box for 'Plot 'y' as:' and change the 'y' to 'lny':



Click 'OK' to close the dialog box.
The points will now be in a straight line.

4) Find the equation of the straight line.

You must use Maths Helper Plus to find this equation accurately as follows:

Display the options dialog for the points as explained in step 3 above. Now click to select the 'Curve Fitting' tab, then click to select the 'linear ☐ y = mx + c' option. (This is second from the top.) Click OK to close the dialog.

The equation of the line through the points will appear on the text view.
Copy this equation carefully, using 'Y' instead of 'y' in your equation because it really means 'lny'.

- For point set 'a', show that the equation of the line is: Y = x + 1.09861

5) Replace ‘Y’ in the equation with ‘lny’.

- For point set ‘a’ we have: lny = x + 1.09861.
- Write this equation in the form $y = ae^{bx}$ or $y = e^{ax+b}$, as shown below:

$$\ln y = x + 1.09861$$

Taking inverse logs both sides:

$$e^{\ln y} = e^{x+1.09861}$$

$$y = e^{x+1.09861}$$ ← If this equation contains only whole numbers, then stop here.

$$= e^{x} e^{1.09861}$$ ← $e^{1.09861} = 2.999993134$ which is rounded to ‘3’.

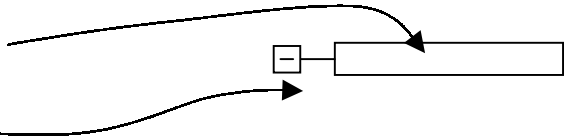
$$= 3e^{x}$$

6) To see if this is correct, plot this equation in Maths Helper Plus. If the points are accurate, the function line will pass through the centre of each one.

(First you will need to reverse the change you made in step 3 above. Double click on a plotted point as before, then change ‘lny’ back to ‘y’.)

To plot the equation you:

- Click on the input box. (On the text view.)
- Type the equation you want to plot.
- Click outside of the input box.



- For point set ‘a’, enter $y = 3e^{x}$ like this: y = 3e^x

[<u>You must</u> put the index in brackets if it has more than one term or factor. E.g.: y = 2e^(3x) or y = e^(x+1)]

If your equation is correct, then the function curve will pass through all points in a given set.

Use the steps described above to find functions that fit each of the other sets of plotted points.
Record your answers in the table below: (Point set ‘a’ has already been completed.)
Graph each function to make sure it passes through each of the original points.

Set of points:	Straight line equation in ‘Y’	Replace ‘Y’ with ‘lny’	Simplify
‘a’ (red dots)	Y = x + 1.09861	lny = x + 1.09861	$y = 3e^{x}$
b (blue **+**)			
c (green circles)			
d (pink dots)			
e (black circles)			